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**A STUDY OF DIGITAL HOLOGRAPHIC
FILTER GENERATION**

by

FRANK INGELS and MALCOLM CALHOUN



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FILTER GENERATION

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Principal Investigator: Frank Ingels
Prepared by: Malcolm Calhoun

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Mississippi State, Mississippi
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SUMMARY

The work statement items of this contract have been addressed in this ~~final~~ report in the following manner:

- A) A thorough bibliography and summary of significant results has been presented in Appendix A of this ~~final~~ report.
- B) A discussion of the more significant problems associated with digital computer generation of holograms has been presented in Chapters I and II.
- C) A criteria for producing optimum digital holograms has been presented in Chapter II. This criteria revolves around amplitude resolution and spatial frequency limitations induced by the computer and plotter process.
- D) Test results have been compiled and examples of the products desired have been created digitally and compound with optically produced products. Chapter IV presents these results.

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CHAPTER I

INTRODUCTION

In classical holography, a recording is made of the interference pattern of an object wave with a reference wave. This system requires the use of a laser, an optical bench, and some recording medium. The binary computer generated hologram proposed by Lohmann and Paris^{12*} presents several advantages over the classical hologram: (1) a physical object need not be used, (2) no laser is required and (3) the requirement for expensive optical equipment is eliminated. In realizing computer generated holograms, however, physical limitations of the computer must be considered.

Storage capacity of a digital computer limits the number of terms in the frequency domain representation of the binary hologram. The device which plots the binary mask for making the hologram is constrained to operate in discrete steps. Thus, the dynamic range of the Fourier transform numbers which are represented in the binary hologram is limited by the digital computer and its associated input-output devices. The problem of producing a binary hologram becomes one of satisfying the physical limitations of the digital computer while maintaining the best possible image reconstruction.

One facet of binary holography that has not been exploited widely is its use in cryptography; that is, the encoding of information in binary holograms.⁸ A cursory study of encoding/decoding binary

*Superscripts apply to entries in the bibliography of Appendix A.

holograms has revealed numerous difficulties in retrieving the encoded information. The alignment of the spatial filters and the related optical equipment is particularly laborious. The problems encountered with the optical system naturally lead to the idea of simulating the optical system with a digital computer as a means of studying information degradation and as a way of optimizing image reconstruction. The fact that lasers, optical benches, and recording media are not required makes computer simulated holography particularly appealing.

The computer simulation process to be used in this study is analogous to a coherent optical processing system¹⁵² capable of realizing operations of the form

$$I(x,y) = K \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\zeta,\eta) h(x - \zeta, y - \eta) d\zeta d\eta \right|^2 \quad (1)$$

where $I(x,y)$ is an intensity distribution, K is a complex constant, $g(x,y)$ is the object function, and $h(x,y)$ is the system impulse response.

A configuration for realizing the intensity distribution $I(x,y)$ is shown in Figure 1. Source S represents a coherent light source and lens L_1 serves as a collimator. The input to be processed is inserted as a space varying amplitude transmittance $g(x,y)$ in plane P_2 . (In the computer simulation, the binary object mask appears at this point). Lens L_2 Fourier transforms $g(x,y)$ producing an amplitude distribution

$$K_1 G(x_2/\lambda f, y_2/\lambda f) \quad (2)$$

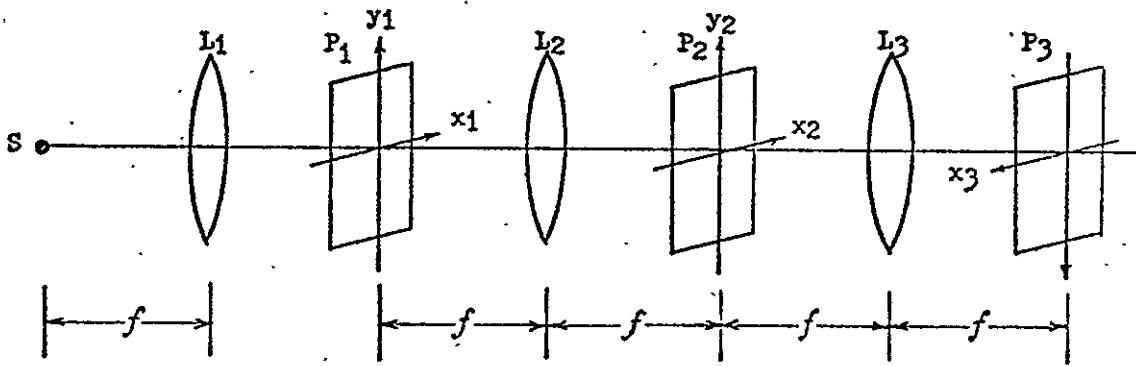


Figure 1. Configuration for Optical Processing System.

at P_2 , the back focal plane of L_2 . K_1 is a complex constant, $G = F[g]$, and λ is the wavelength of the source.

The amplitude and phase of the frequency domain distribution at P_2 may be manipulated by the insertion of a frequency plane filter with amplitude transmittance

$$t(x_2, y_2) = K_2 H(x_2/\lambda f, y_2/\lambda f) \quad (3)$$

where $H = F[h]$. Thus the amplitude distribution to the right of P_2 is simply the frequency domain product GH . The lens L_3 Fourier transforms GH to yield an intensity distribution at P_3

$$I(x_3, y_3) = K \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\zeta, n) h(-x_3 - \zeta, -y_3 - n) d\zeta dn \right|^2 \quad (4)$$

where the minus signs preceding x_3 and y_3 are a consequence of

$$F[F[g(x, y)]] = g(-x, -y) \quad (5)$$

In particular, if the transmittance of the frequency domain filter at P_2 is $t(x_2, y_2) = 1$, then $GH = G$ and lens L_3 simply reconstructs the original image function from P_1 with the space coordinates reversed.

This sign problem is resolved easily by defining the coordinate system x_3 and y_3 as shown in Figure 1.

A particular application of the optical processing system of Figure 1 is in the area of character recognition.¹⁵² A specific spatial signal $s(x, y)$ is inserted into the system at P_1 . A matched filter is then inserted at P_2 . By definition, a linear space invariant filter is said to be matched to a signal $s(x, y)$ if its impulse response $h(x, y)$ is given by $h(x, y) = s^*(-x, -y)$. For an input signal $g(x, y)$ applied to a matched filter $h(x, y)$, the output $v(x, y)$ is

$$v(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\zeta, \eta) h(x - \zeta, y - \eta) d\zeta d\eta \quad (6)$$

$$v(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\zeta, \eta) s^*(\zeta - x, \eta - y) d\zeta d\eta \quad (7)$$

which is just the cross correlation of g and s . Fourier transformation of the impulse response of the matched filter requires that

$$F[h(x, y)] = F[s^*(-x, -y)] \quad (8)$$

$$\text{or} \quad H(f_x, f_y) = S^*(f_x, f_y) . \quad (9)$$

Note that * denotes the complex conjugate. Thus the frequency plane mask at P_2 must have amplitude transmittance proportional to S^* . The

field distribution transmitted by the mask is then proportional to

$$\int_{-\infty}^{\infty} \int s(\zeta, \eta) s^*(\zeta - x, \eta - y) d\zeta d\eta = SS^* \quad (10)$$

which is entirely real. This implies that the frequency plane mask cancels all curvature of the wave front yielding a plane wave. This plane wave front is then brought to a bright focus at P_3 by the lens L_3 . Thus the presence of a signal may be detected by measuring the intensity of light at P_3 as a function of input signal s . Optical interpretation of the matched filtering operation is shown in Figure 2 and the output of the processor for a cross correlation application is shown in Figure 3. W_h and W_g are the spatial width of h and g respectively.

Computer Generated Holograms

Generally, computer generated holograms are capable of functioning just as optically formed holograms. The following is a discussion of some of the basic principles of computer generated holograms.

The Whittaker-Shannon sampling theorem states that it is possible to compute a limited number of samples of a continuous function and with these samples to reconstruct the continuous function exactly.¹⁵⁶ The preceding theorem is subject to the constraints that (1) the continuous function is band-limited and (2) the continuous function is sampled at least twice in any increment as large as the spatial period of the highest spatial frequency in the function. Thus, it is possible to obtain a Fourier transform representation of an appropriately

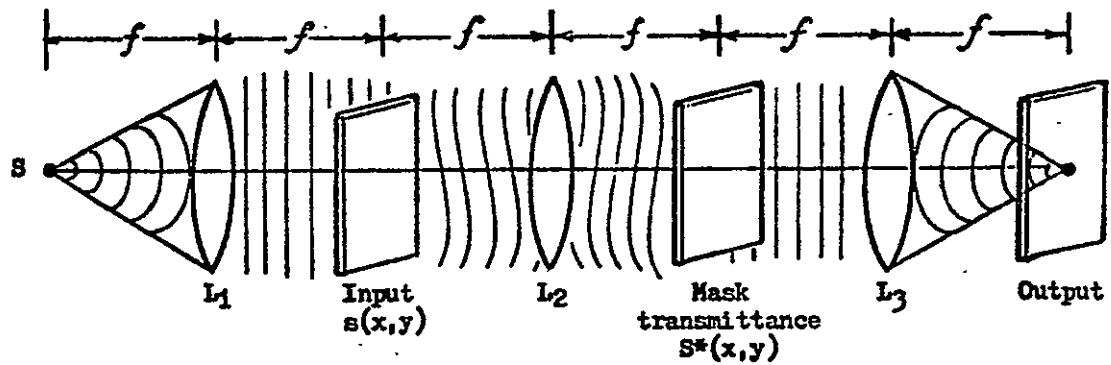


Figure 2. Optical Interpretation of the Matched Filtering Operation.

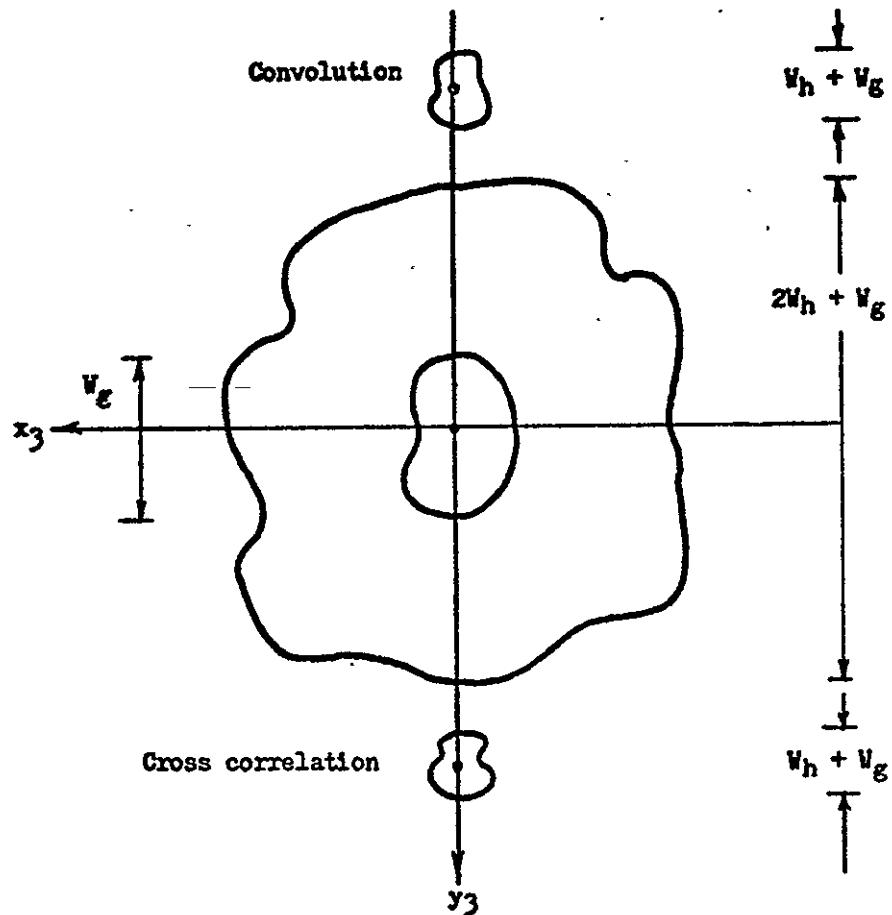


Figure 3. Location of Various terms of the Processor Output with a Matched Filter.

sampled object function. In realizing a computer generated hologram, the computer must calculate a great many samples of the two-dimensional Fourier transform of the subject. In making binary holograms by means of a computer, the transmittance is considered to be zero or one. The transmittance of the binary mask representing the subject is also one or zero.

In order to understand how the computer can create a hologram which is comparable to an optically produced hologram, it is necessary to consider the mathematics of sampling and of the discrete Fourier transformation. Mathematically, sampling may be accomplished by means of the comb function; defined as

$$\text{comb}(x) = \sum_{n=-\infty}^{\infty} \delta(x - n) \quad (11)$$

where $\delta(x)$ is the unit impulse function and n is an integer. See Figure 4.

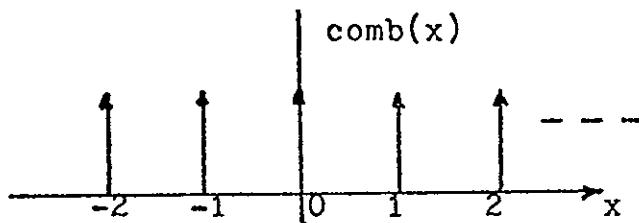


Figure 4. The Comb Function

Furthermore, the Fourier transform of the comb function,

$$F[\text{comb}(x)] = \text{comb}(f) . \quad (12)$$

The sampling operation may be represented as a multiplication of a continuous function $g(x)$ by the function

$$\sum_{n=-\infty}^{\infty} \delta(x - n\Delta x) = \frac{1}{\Delta x} \text{comb}(x/\Delta x) \quad (13)$$

where Δx is the sampling interval.¹⁵⁶ Thus, the sampled function $g_s(x)$ may be expressed as

$$\begin{aligned} g_s(x) &= g(x) \cdot \frac{1}{\Delta x} \text{comb}(x/\Delta x) \\ &= g(x) \sum_{n=-\infty}^{\infty} \delta(x - n\Delta x) \\ &= \sum_{n=-\infty}^{\infty} g(n\Delta x) \delta(x - n\Delta x) \end{aligned} \quad (14)$$

Each sample of $g(x)$ is a delta function whose strength is given by the value of $g(x)$ at the position of the delta function as shown in Figure 5.

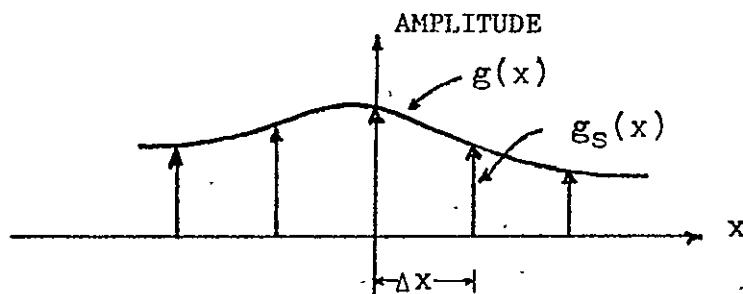


Figure 5. The Continuous Function $g(x)$ and the Sampled Function $g_s(x)$

Computer generated holograms require computation of the Fourier transform $G_s(x)$ of the sampled spatial function $g_s(x)$. If the sampling has been carried out in accordance with the sampling theorem, the Fourier transform $G(f)$ may be obtained from $G_s(f)$. The original spatial function is simply the desired image reconstruction and may be accomplished optically or mathematically, the latter being the approach used in the proposed simulation process.

In transforming $g_s(x)$, the product $g(x)$ and $\frac{1}{\Delta x} \text{comb}(x/\Delta x)$ in the spatial domain becomes the convolution of their Fourier transforms in the frequency domain.

$$\begin{aligned} G_s(f) &= G(f)^* \text{comb}(\Delta x f) \\ &= G(f)^* \frac{1}{\Delta x} \sum_{n=-\infty}^{\infty} \delta(f - n/\Delta x) \end{aligned} \quad (15)$$

From the definition of convolution,

$$G_s(f) = \frac{1}{\Delta x} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} G(u) \delta(f - n/\Delta x - u) du \quad (16)$$

$$G_s(f) = \frac{1}{\Delta x} \sum_{n=-\infty}^{\infty} G(f - n/\Delta x), \quad (17)$$

as shown in Figure 6.

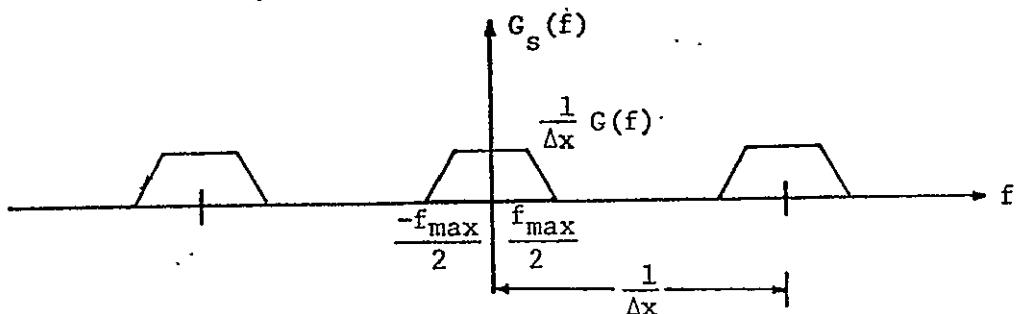


Figure 6. Fourier Transform $G_s(f)$ of the Sampled Spatial Function.

If the original space function $g(s)$ is bandlimited so that $G(f)$ has non-zero values only in the interval

$$-\frac{f_{\max}}{2} \leq f \leq \frac{f_{\max}}{2} \quad (18)$$

overlap of the shifted transform is prevented as long as $\frac{1}{\Delta x} \geq \frac{f_{\max}}{2}$ is satisfied. Thus, $\Delta x \leq 1/f_{\max}$ specifies the conditions of the Whittaker-Shannon sampling theorem.

To recover $G(f)$ from $G_s(f)$, the sampled Fourier transform is simply multiplied by a rectangular window function $\Delta x \operatorname{rect}(f/f_{\max})$,

$$G(f) = G_s(f) \Delta x \operatorname{rect}(f/f_{\max}) \quad (19)$$

In order to recover the original spatial function $g(x)$, it is necessary to take the inverse Fourier transform of the product $G_s(f) \Delta x \operatorname{rect}(f/f_{\max})$. This is equivalent to convolving in the spatial domain the inverse Fourier transforms of these two functions. The inverse Fourier transform of the window function is given by

$$F^{-1}[\Delta x \operatorname{rect}(f/f_{\max})] = \Delta x f_{\max} \frac{\sin(\pi f_{\max} x)}{\pi f_{\max} x} \quad (20)$$

Thus,

$$\begin{aligned} g(x) &= F^{-1}[G(f)] = g_s(x)^* \Delta x f_{\max} \frac{\sin(\pi f_{\max} x)}{\pi f_{\max} x} \\ &= \sum_{n=-\infty}^{\infty} g(n\Delta x) \delta(x - n\Delta x)^* \Delta x f_{\max} \frac{\sin(\pi f_{\max} x)}{\pi f_{\max} x} \\ g(x) &= \Delta x f_{\max} \sum_{n=-\infty}^{\infty} g(n\Delta x) \frac{\sin \pi f_{\max} (x - n\Delta x)}{f_{\max} (x - n\Delta x)} \end{aligned} \quad (21)$$

The preceding discussion may be extended to two dimensional spatial functions $g(x,y)$ by replacing the comb function with a two dimensional array of delta functions, $\text{comb}(x) \text{ comb}(y)$.

In practice, the Fast Fourier transform algorithm is used for computing the Fourier transform of the object function. In the production of computer generated holograms, the Fourier transformed function representation is plotted and photographed to create a binary Fourier transform hologram. It is worth noting that the binary hologram is a special form of spatial filter.

CHAPTER II

DEFINITION OF THE PROBLEM

The primary sources of problems in generating synthetic holograms are the physical limitations of the digital computer and its peripheral equipment. The main objectives of this research are to (1) study the effects of computer induced degradations on binary holograms, (2) develop a technique for optimizing reconstruction of images by computer generated holograms, and (3) verify the optimization process by observing image reconstructions, both optically and by computer simulation.

Some knowledge of the Lohmann-type hologram is necessary in order that the problems of generating holograms by means of the computer be fully appreciated. Lohmann holograms are binary in nature and have a non-negative, real amplitude transmittance.¹² The Fraunhofer effect is achieved by means of the so called "detour phase." The synthetic hologram produced by this procedure is in effect a diffraction grating which will reproduce an image when illuminated by a coherent, monochromatic light source. Image reconstruction from a Fraunhofer hologram is illustrated in Figure 7. The Fraunhofer diffraction pattern in the image plane is described by the function

$$U(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(f_x, f_y) e^{-j2(\frac{f_x}{\lambda f} + \frac{f_y}{\lambda f})} df_x df_y ; \quad (22)$$

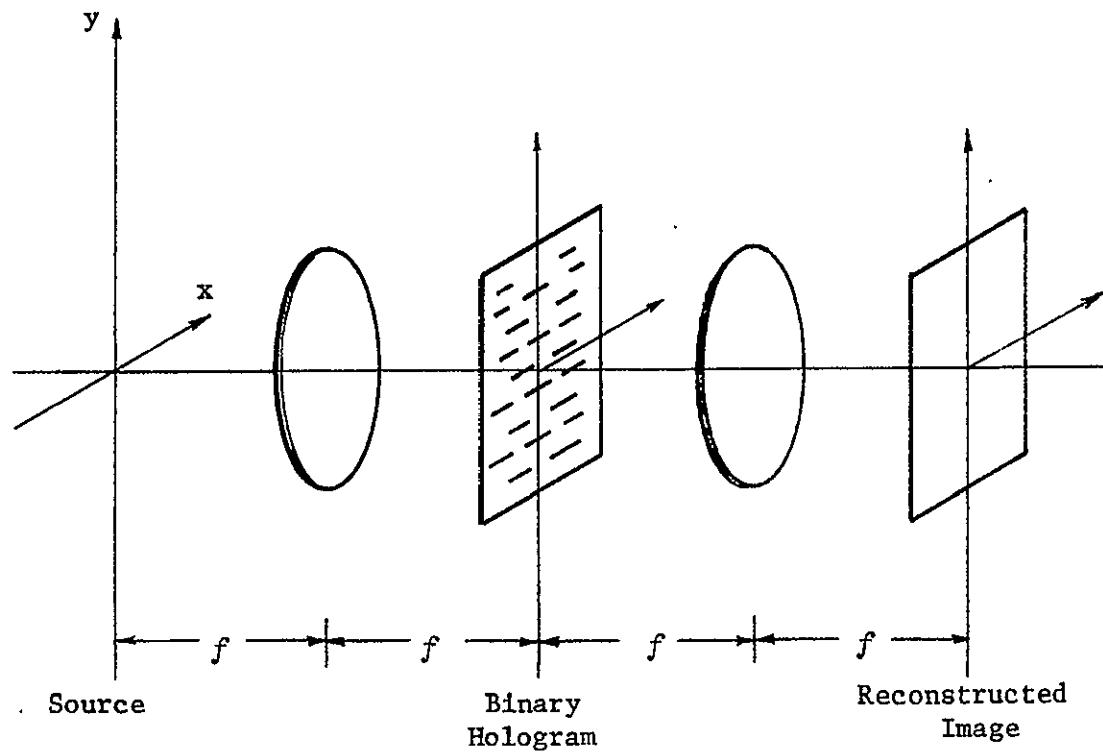


Figure 7. Optical Setup for Reconstruction of Binary Hologram.

λ represents the wavelength of the incident light and f is the focal length of the lens. The problem becomes that of synthesizing the complex function

$$F(f_x, f_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x, y) e^{j2\pi(\frac{f_x}{\lambda f}x + \frac{f_y}{\lambda f}y)} dx dy \quad (23)$$

which may be recognized as the inverse of the two-dimensional Fourier transform of equation (22). Lohmann and Paris have presented a method of synthesizing binary synthetic Fraunhofer holograms capable of realizing the image $U(x, y)$. Of course, there are some constraints placed on the image, limiting both its size and number of frequency terms.

Limitations on image size require a finite extent $\Delta x = \Delta y$ in area. The minimum resolvable region is limited to $\delta x = \delta y = 1/\Delta x$. The number of resolvable points in the image is then given by

$$N^2 = \frac{\Delta x \Delta y}{\delta x \delta y} = \left(\frac{\Delta x}{\delta x} \right)^2 \quad (24)$$

The number N^2 is known as the space-bandwidth product and is also equal to the number of cells in the binary hologram. Each cell of the Lohmann hologram acts as a miniature diffraction grating; its geometry is shown in Figure 8.

Since both the object and the reproduced image are finite in extent, the two-dimensional Fourier transform of equation (22) may be replaced by a two-dimensional discrete Fourier transform of N^2 terms. The object to be described is divided into $N \times N$ regular grids with spacing $\delta v = 1/\Delta x$. Each of these space samples will have a complex

frequency term $F(m\delta\nu, n\delta\nu)$. In turn, this complex number will be recorded as dictated by Lohmann's algorithm to produce a synthetic binary hologram.

The complex frequency domain number representation is recorded in the Lohmann cell of Figure 8. The magnitude of the transmittance is determined by the size of the cell aperture, $W_{nm}\delta\nu$. In practice, this is the only magnitude parameter allowed to vary, $C\delta\nu$ remaining fixed at approximately 1/2 the cell size for maximum image intensity. The phase of the complex frequency term determines the position of the aperture, $P_{nm}\delta\nu$. The cell can shift from $-\pi$ to π , modulo 2π , with the center of the cell representing zero phase.

At Mississippi State University programs have been developed for generating Lohmann-type holograms using a Univac 1106 computer and a Gould electrostatic plotter. Cell realization is based on the ability of the plotter to plot 80 dots per inch in two dimensions. The field chosen for the synthetic hologram is 10 inches by 10 inches, thus the plotter provides an array of 800 by 800 dots for each hologram produced. The cell realized by the Gould plotter is shown in Figure 9.

The physical extent of each Lohmann cell is limited by the number of frequency terms in the hologram. In this study, the sample space is 64 by 64 and the corresponding maximum number of frequency terms is the same. Thus, a cell size may vary from $800/64 = 12$ dots to a maximum size of 800 dots for a hologram containing only one frequency term. Again, it should be noted that the cells are square by design. Also observe that in recording the 64 frequency terms in 800 dots the number of dots per cell is 12 due to the plotter quantization.

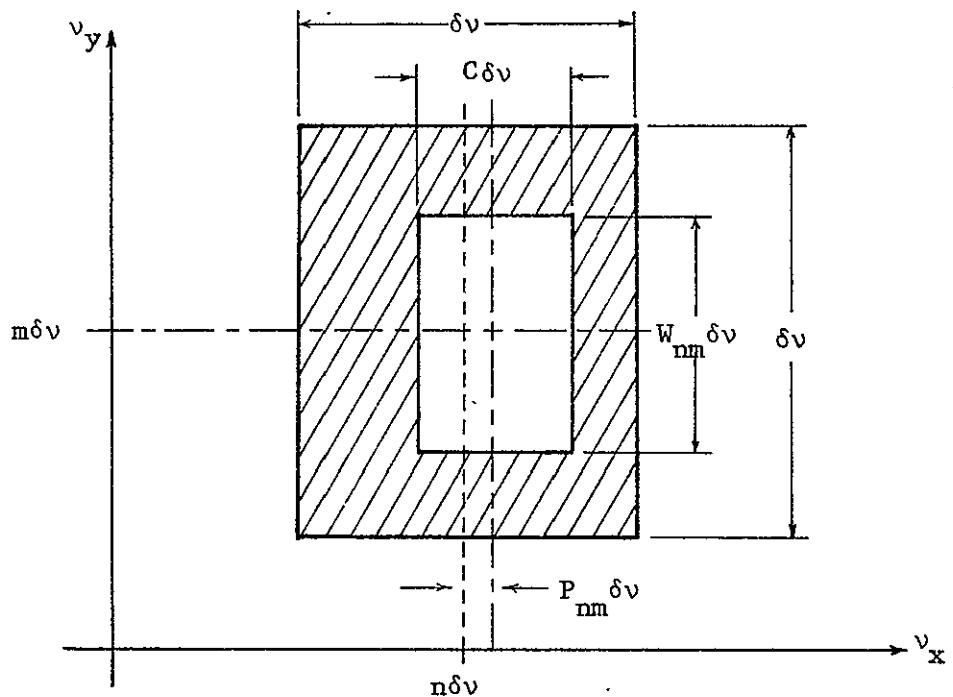


Figure 8. Lohmann Cell Geometry.

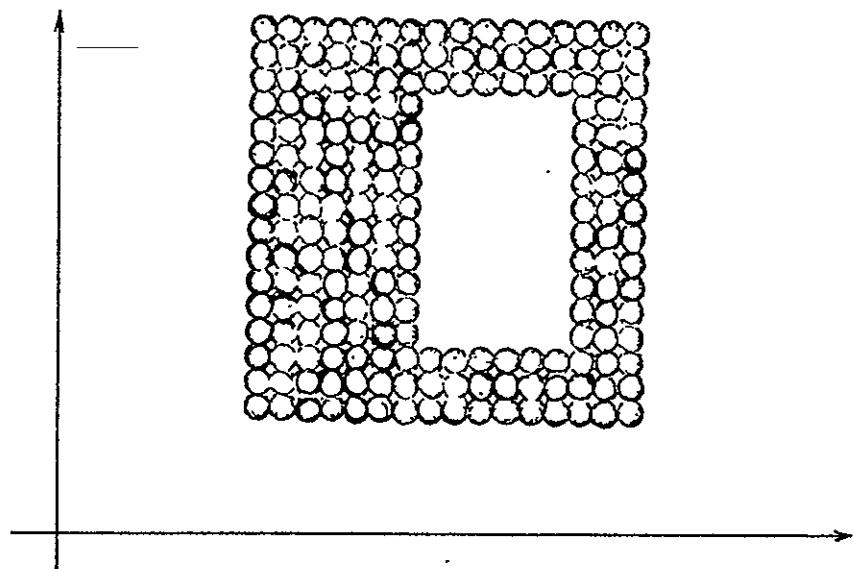


Figure 9. Gould Plot of Lohmann Cell.

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The smallest change that can be made in the amplitude representation in the case of 64 frequency terms is $1/12 = 8.33\%$ quantization in amplitude. For a hologram recording with only 40 frequency terms, the cell size will be $800/40 = 20$ dots resulting in a quantization error of only 5%. Thus, the smaller the number of frequency terms, the larger the cell and the less the quantization error.

Phase quantization may be regarded in a similar manner. In the example of 64 frequency terms and 12 dots per cell, the phase quantization is $2\pi/12 = \pi/6$ or 30° phase quantization steps. Reduction to 40 frequency terms and the corresponding 20 dots per cell yields phase quantization of $2\pi/20 = 18^\circ$. Again, larger cells reduce the quantization error.

Generating the hologram becomes a problem in terms of the dynamic range of the frequency terms. The higher frequency terms are usually small in amplitude, thus much of their information content is lost in quantization steps. One technique for reducing dynamic range is to band limit the number of frequency terms. In this method care must be exercised not to sacrifice information contained in the higher frequency terms for the sake of improved resolution.

A second method of reducing dynamic range of the hologram is to amplitude limit the low frequency terms of the Fourier transform numbers. Since a binary mask resembles an array of rectangle functions, the Fourier transform of each point is basically a sinc function. The zeroth order term is usually large in amplitude with the higher frequency terms decreasing rapidly. The low frequency, large amplitude

terms may be clipped to reduce the dynamic range of the hologram recording without appreciable loss of information.¹³⁴

In order to obtain the best possible image reconstruction within the limits of the digital computer, some image quality measurement must be defined. For this investigation, an integral of signal amplitude squared over the reconstruction aperture has been chosen.¹³⁴ This technique relates directly to intensity error and is easily adaptable to digital computer analysis.

This error measurement entails a comparison of the pure reconstruction image to the image aperture which has been clipped and band-limited. The algorithm for determining the numerical error introduced by clipping and bandlimiting first Fourier transforms the object aperture under investigation; the resulting frequency domain function $g(f_x, f_y)$ is again Fourier transformed to produce $I(x, y)$, the space domain image with no degradation. The frequency domain expression $\underline{g(f_x, f_y)}$ is also operated on by amplitude clipping and bandlimiting to reduce its dynamic range. The corrupted array $g_c(f_x, f_y)$ is then returned to the spatial domain as $I_c(x, y)$. The two resulting arrays are then term by term compared, squared, and summed over the image aperture to yield the error figure. A block diagram of the error algorithm is shown in Figure 10.

Since the error criterion is based on image intensity, only the amplitude of the image apertures need to be considered. To avoid the possibility of scale differences in the two reconstructed images being compared, a scaling factor has been used in the error analysis algorithm. The discrete error function including the scale factor s

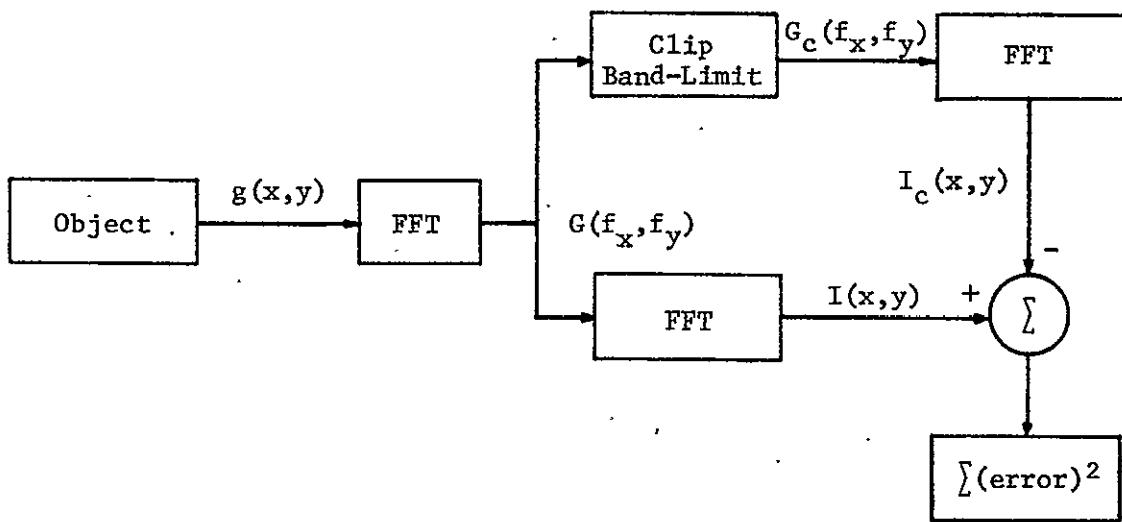


Figure 10. Block Diagram of Error Analysis Algorithm.

may be written

$$\sum_m \sum_n (|I_{mn}| - s|I_{cmn}|)^2 \quad (25)$$

where I_{mn} and I_{cmn} are the pure image and the degraded image respectively over the discrete image aperture which is m by n . The problem of finding a minimum of the error function in terms of the parameter s is solved by expanding the squared quantity under the double summation, differentiating with respect to s , then setting the resulting expression to zero to solve for s . The resulting value of s which minimizes the error function is

$$s = \frac{\sum_m \sum_n |I_{mn} \cdot I_{cmn}|}{\sum_m \sum_n I_{cmn}} \quad (26)$$

Finally, to obtain a signal to noise figure for the corrupted image, the error figure is divided into the pure image power

$$\sum_m \sum_n |I_{mn}|^2 \quad (27)$$

The signal to noise figure for a given set of clipping and band-limiting parameters is given by

$$SN = \frac{\sum_m \sum_n |I_{mn}|^2}{\sum_m \sum_n (I_{mn} - sI_{cmn})^2}, \quad (28)$$

where the image I_c is a function of the clipping and bandlimiting parameters chosen to satisfy computer constraints.

One of the major efforts of this research is a computer program for determining an optimum set of parameters to be used in producing a synthetic hologram. The optimizing routine employs the signal to noise criterion described above. The function to be optimized is a non-linear function of two variables with an additional constraint that the bandlimiting parameter be an integer number. Optimizing a function of this complexity is a formidable problem, and according to Dr. R. R. Hocking,* authority on non-linear programming techniques, "there is no guarantee that an absolute maximum or minimum of such a function can be located." A sequential search technique with a post-optimum search was eventually employed with success.

Simulated hologram reconstruction was implemented because of the difficulty of achieving good optical reconstructions with limited

*Private communication with Dr. R. R. Hocking, Professor of Mathematics and Statistics, Mississippi State, MS, 39762.

optical facilities. Simulated reconstructions allow images to be observed and signal to noise optimization routines to be studied without the usual photography, laser, and optical paraphernalia.

The simulation program functions as a two lens optical system as shown in Figure 11. A flow chart for the simulation program is given in Figure 12. This routine may be used for various spatial filtering experiments as well as simple image reconstruction. For example, matched filtering and autocorrelation may be simulated by this algorithm.

One of the major efforts of this research has been the recovery of the holograms from the tape which is used to drive the plotting equipment. The information recorded on the tape has been clipped and bandlimited as previously described. In addition, it has suffered quantization effects due to the discreteness of the plotter. Thus, the recovery of this Fourier domain array makes available a simulated image with all of its degradation effects. The flow chart for this algorithm is in Figure 13.

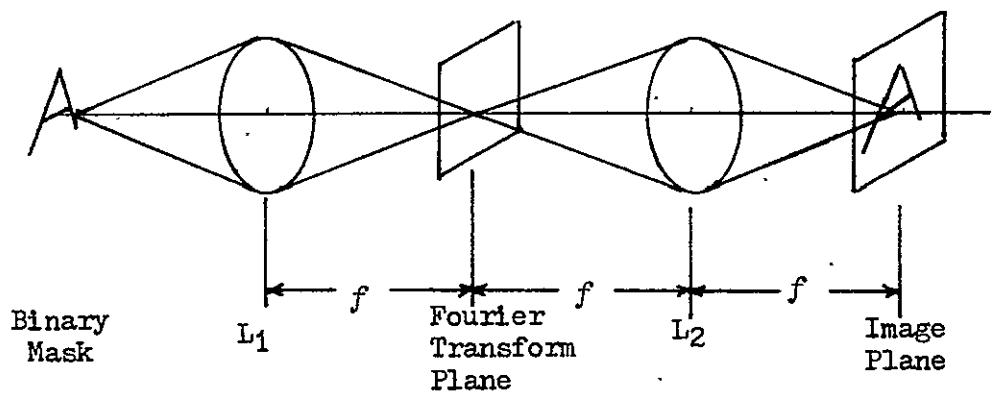


Figure 11. Optical System with Two Lenses.

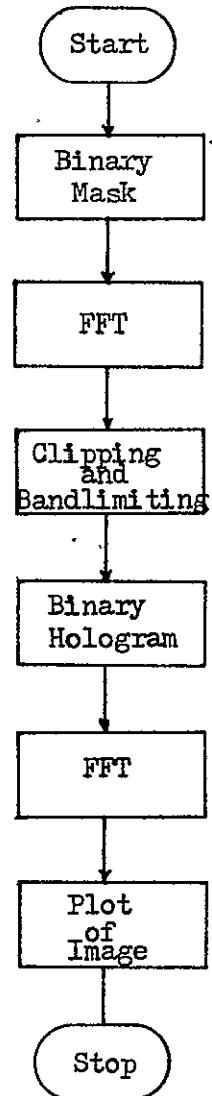


Figure 12. Computer Simulation of Two Lens Optical System.

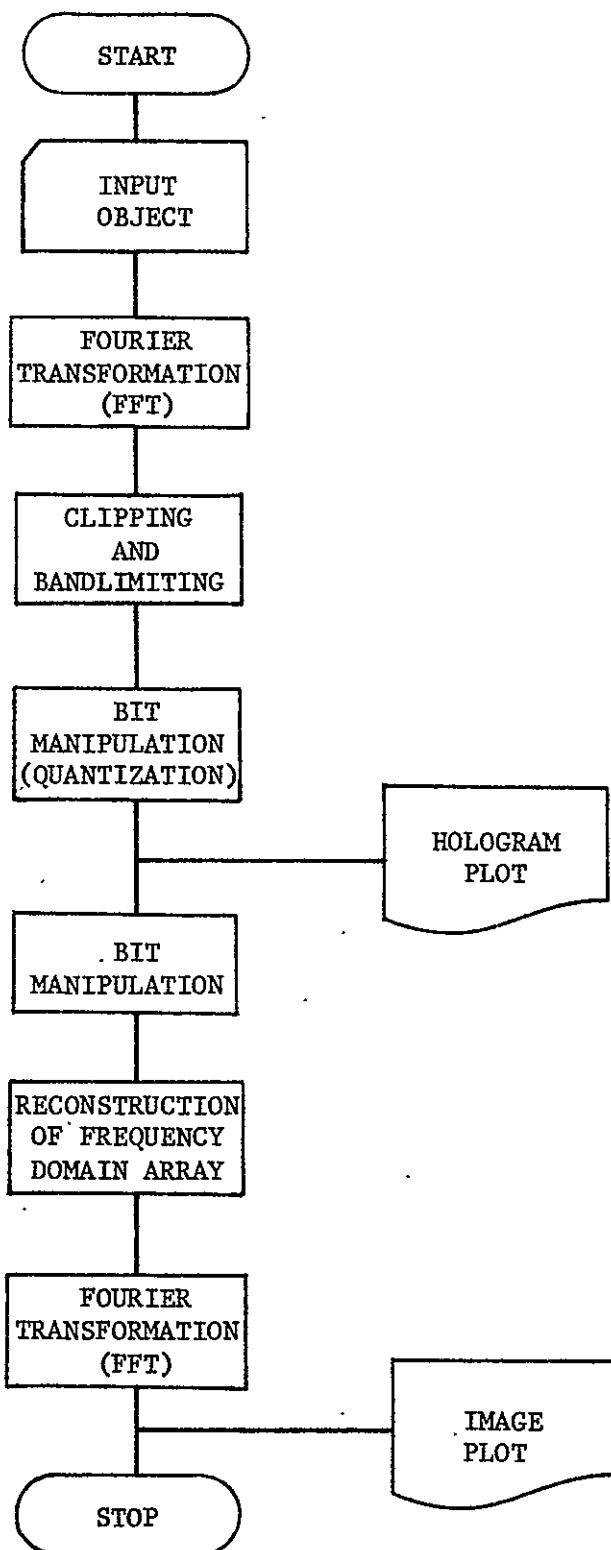


Figure 13. Flow Chart of Reconstruction Program.

CHAPTER III

A SYNOPSIS OF DIGITAL COMPUTER PROGRAMS

The Univac 1106 Computer at the Mississippi State University Computing Center was used for the development of all holograms, spatial filters, and simulation plots in this study. Thus, it is necessary to describe the programs and to explain their usage. The majority of the programs are written in Fortran V, the exceptions being those programs directly related to the Gould plotter which are written in Univac 1106 assembler language. A listing of all programs and subroutines used in this study is given in Appendix B.

There are some routines which are employed in virtually all computer runs; these programs are on file and are called as subroutines as required by a main program. The commonly used subroutines will be discussed first.

The spatial dimension of the object description utilized in this study was chosen to be a 64 by 64 square array. This necessitates a complex dimensioning of 64 by 64 storage arrays in the computer. Complex specification is required because the Fourier transformation of the binary transmittance masks produces complex numbers. This study is restricted to two-dimensional arrays although it could be extended to include three-dimensional cases. Because the primary concern in this study is information storage and processing, the two-dimensional analysis is sufficient.

Subroutine FFT2(A,M)

FFT2 is a two dimensional Fast Fourier transformation based on the Cooley-Tooley algorithm. The input matrix A is a M by M array of complex numbers in rectangular form. M may be any integer power of two up to a maximum value of 64. The ordering of the elements of A requires that A be operated on by subroutine ROTATE prior to insertion into FFT2. The matrix A is returned as the Fourier transformation of the input matrix.

Subroutine ROTATE(A,B,M)

ROTATE takes an M by M complex array A as read in row-column order of a spatial coordinate system and rotates it into matrix B which is in the order required by FFT2. Henceforth, a reference to a rotated matrix implies that a matrix has been operated on by ROTATE and is in the order expected by FFT2. The input matrix A remains unchanged.

Subroutine POLAR(A,B,M)

This subroutine takes a M by M complex array A which is in rectangular form ($a + jb$) and converts it to the M by M matrix B in polar form. The magnitude of B is stored as the real component of B while the angle appears as the imaginary part of B. Matrix A is unchanged.

Subroutine BUILD(A,B,NQ,M)

This program functions as a bandpass filter and limits the output matrix B to NT terms, where NT is less than or equal to 64. The output is matrix B, NT by NT, complex, and un-rotated. NT is simply the

desired bandpass and is a function of the input parameter NQ. NQ is related to the number of quanta in the Lohmann cell created by the plotter. NT is found from an array QT in subroutine BLOCK and common QUAN to be described later. The input matrix A remains unchanged. The output matrix B is used by subroutine BITS to produce the binary hologram.

Subroutine BITS(A,CLIP,NQ,NT2,B)

BITS takes the bandlimited matrix A from BUILD, and with a specified clipping level CLIP, forms the 800 by 800 bit array used to produce the binary hologram. The input matrix A must be in polar form. The array of terms is NT by NT, where NT is determined by

$$NT = QT(1, NQ) \quad (29)$$

The cell size is found from the same data block and the parameter NQ by

$$NC = QT(2, NQ) \quad (30)$$

The resulting phase quantization is $2\pi/NC$ radians and the aperture width is set for optimum intensity as shown in Figure 8 to be $NC/2$ or $C = 1/2$. Note that the subroutine argument NT2 is used to specify a positive or a negative type of plot; NT2 = 1 plots a black background with transparent apertures while NT2 = 2 causes the background to be white with opaque apertures. The output matrix B is simply a one-dimensional 4600 term array which is a portion of the bits used in making the hologram.

Subroutine INIT(N2,NC)

This is an assembler language routine which is called by BITS to initialize subroutine BS to a specified cell size NC and aperture width N2.

Subroutine BS(B,IWL,IMAG,ISH)

BS is another assembler language program used by BITS. Each time BITS calls BS one of the NT by NT bit cells of the hologram is generated by bit manipulation. The argument list is specified by BITS and is of no concern to the user.

Subroutine BITS2(A,CLIP,NQ,N12)

This subroutine is used in forming the plot of the reconstructed image in the simulation program. The input matrix A should be in polar form.

Subroutine CRUPT(B,C,CLIP,NQ,N)

CRUPT takes the frequency domain matrix B, which is in polar rotated form and corrupts it as prescribed by parameters CLIP and NQ. The argument N is 64 as set by the maximum number of terms in the complete spatial array. The returned matrix C is NT by NT terms as determined by the argument NQ. C is returned in rectangular, rotated form. The matrix B remains unchanged.

Subroutine OPT(A,B,F,NQ,PCT,X)

This subroutine finds optimum clipping and bandlimiting values of the object aperture to be made into a hologram. The matrix A is

64 by 64 and has been Fourier transformed twice. It is in polar form. The matrix B is in polar form, has been Fourier transformed once, and is 64 by 64. Matrix F is returned as an array of signal to noise figure while X is the optimum signal to noise figure. NQ and PCT are returned as optimum values for achieving the best signal to noise ratio in the synthetic hologram. See Figure 14.

Subroutine PIC(A,CLIP,NQ,N)

This program plots a nomograph of the amplitude limited array which was used to produce the hologram. The input matrix A is in polar, un-rotated form and is N by N in size. CLIP and NQ are used as in previous subroutines.

Subroutine TAPE(L,\$)

This subroutine reads the bits from Fortran File 9 that are used in making the actual plot of the hologram.

Function FMAX(A,N)

The maximum value of a matrix A is found by this function subroutine. The matrix A is N by N in size and in polar form.

Function ERR(A,B,ASQ,CLIP,NQ,N)

The error analysis previously described is accomplished by this program. Matrix A is the uncorrupted N by N reference image I in polar form. Matrix B is N by N, polar, and uncorrupted at its time of entry into ERR. CLIP and NQ apply as before to clip and bandlimit

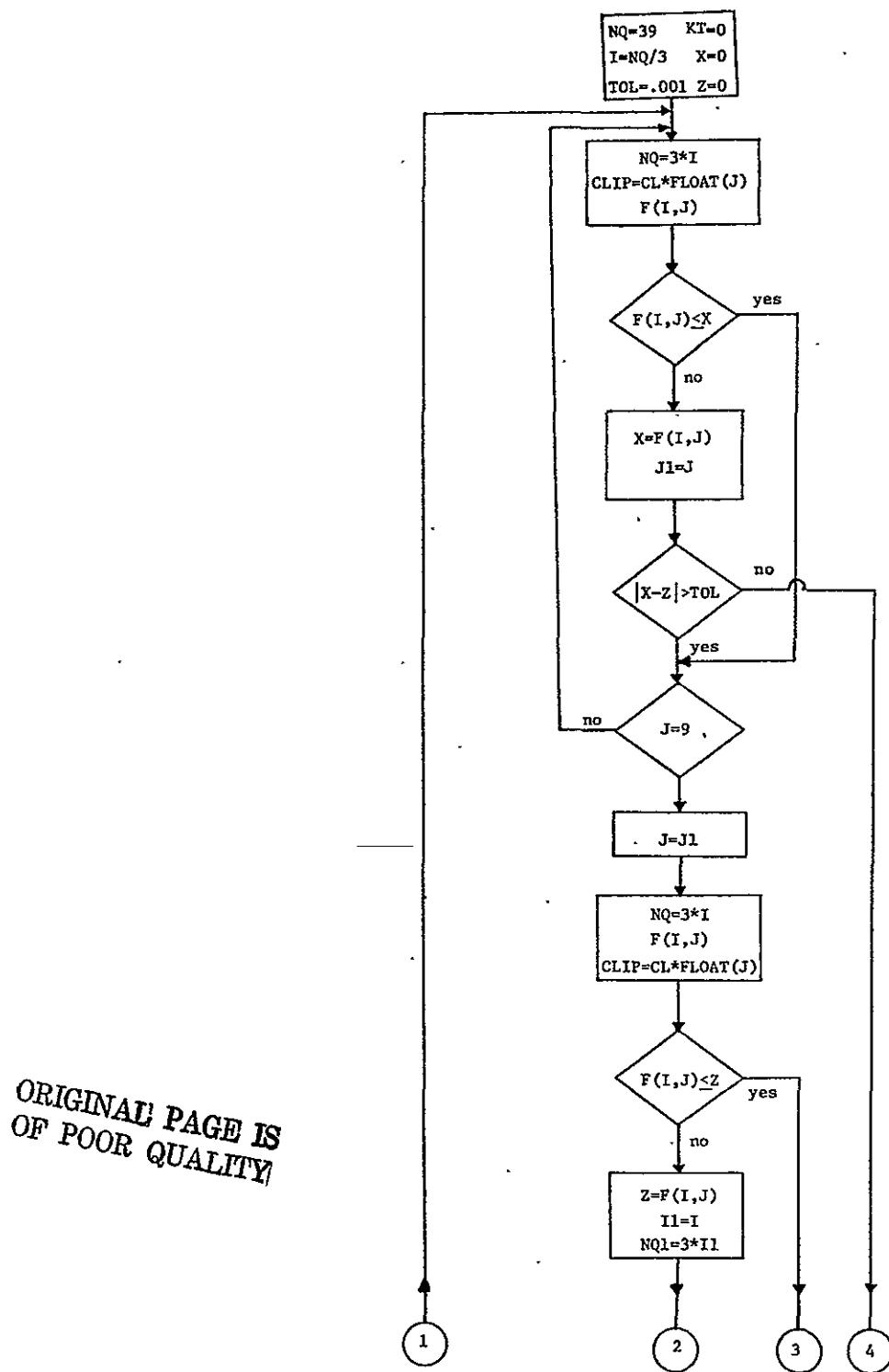
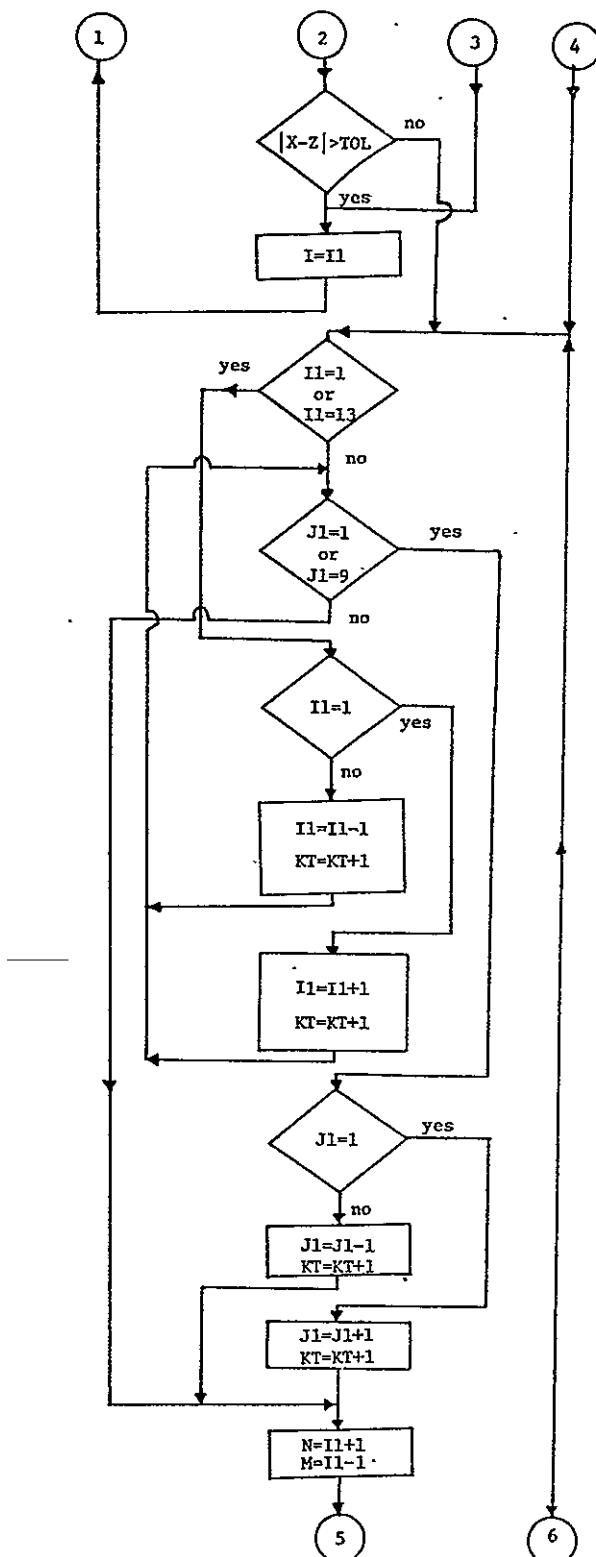
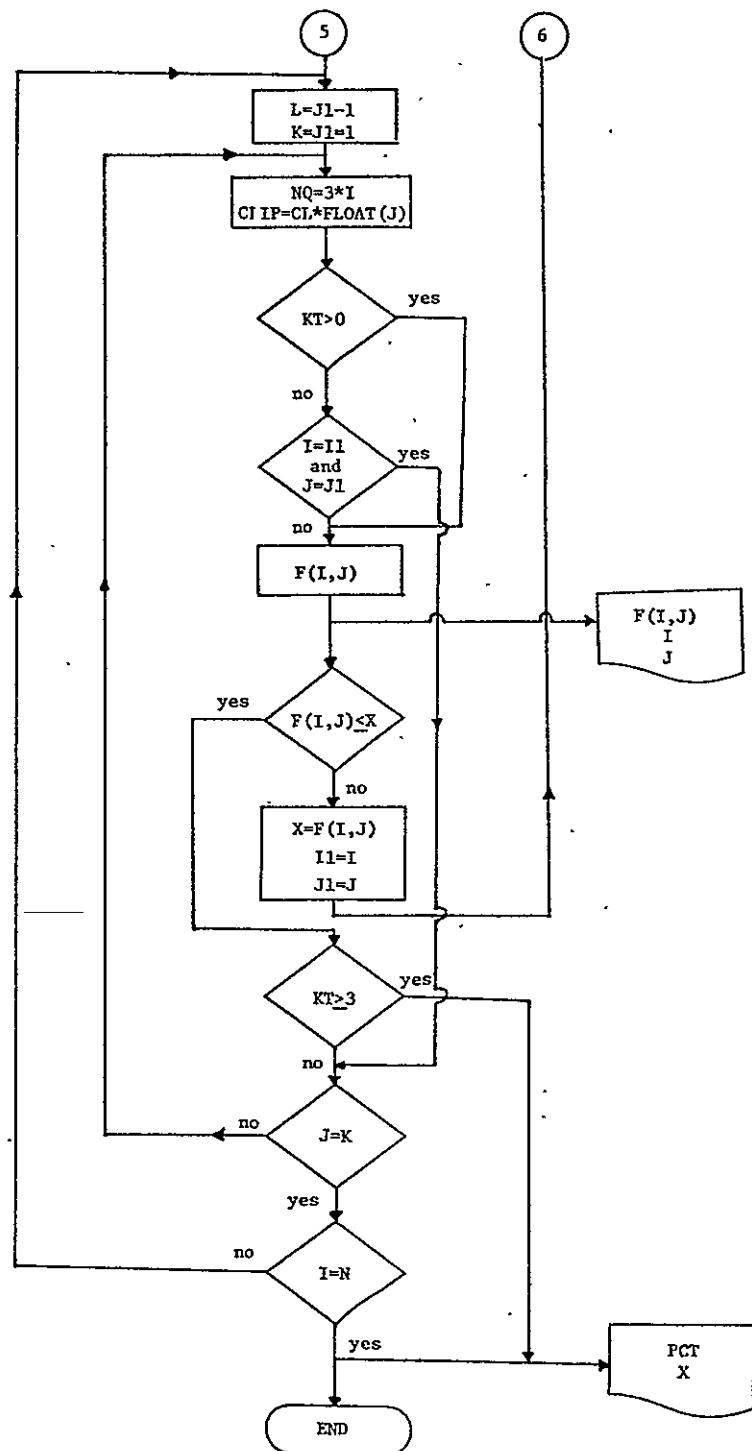


Figure 14. Flow Chart for Optimization Program OPT.



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Figure 14. (Continued)



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Figure 14. (Continued)

matrix B. ASQ is the reference image power that is used in the signal to noise figure calculation.

Block Data BLOCK

BLOCK initializes the named common QUAN which contains the QT array. It is from QT that the bandwidth NT and the cell size NC are found as given by equations (29) and (30) respectively.

Main Program HOLPLO

This program takes the 64 by 64 array of binary numbers used to describe an object and creates a synthetic hologram. A flow chart for this program is given in Figure 15. HOLPLO makes use of several of the previously listed subroutines.

Main Program SIMUL

SIMUL is the Fortran program which simulates the two lens optical system. Its flow chart is given in Figure 12. The plotted output of SIMUL is the image reconstruction of the synthetic hologram created by HOLPLO.

Main Program OPT1

OPT1 takes the object description which is to be made into a hologram and determine optimum clipping and bandwidth values for it. The outputs NQ and PCT from subroutine OPT may be inserted directly into HOLPLO for production of a near optimal hologram.

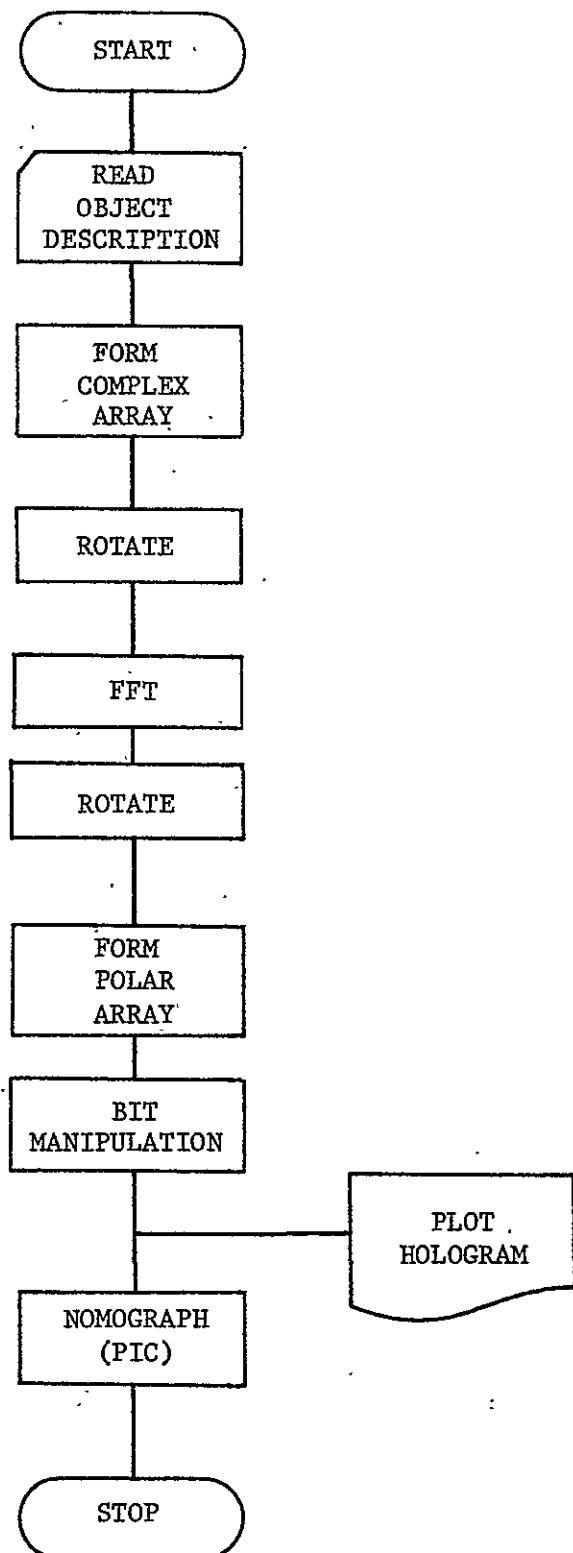


Figure 15. Flow Chart of the Synthesis of a Computer Generated Hologram

Main Program OPT2

This program functions in the same manner as OPT1 except that it finds the optimum value of an encoded hologram.

Main Program SN

Program SN varies the clipping levels and the number of frequency terms in the Fourier transformed array to provide an output array of signal to noise figures as functions of variables NQ and PCT. Main program OPT1 supercedes SN and eliminates the need for unnecessary and lengthy calculations.

Main Program RECON

RECON is used in conjunction with HOLPLO to recover the hologram from the tape which drives the plotter. In ordinary usage, RECON follows HOLPLO sequentially and provides a simulated image reconstruction which includes clipping, bandlimiting, and phase and amplitude quantization induced by the plotter. See Figure 13 for a flow chart of RECON.

Main Program SNPLOT

SNPLOT is a routine for calculating the signal to noise ratio of the hologram after it has been degraded by the plotter. It is run sequentially after HOLPLO.

Main Program SPAFIL

SPAFIL is used for simulating an optical spatial filtering processor. The inputs are descriptions of objects which are compared with digitally generated spatial filters. The output is a plot of the simulated autocorrelation. This program may be used for other spatial filtering applications.

Main Program CODE

CODE is a program which modifies HOLPLO for creating an encoded computer generated hologram. Main program OPT2 is employed to find optimum values for NQ and PCT. A SIMUL type program is used for recovering the encoded information and plotting the reconstruction.

CHAPTER IV

PRESENTATION OF RESULTS

The synthetic holograms produced in this research effort are made by (1) electrostatically plotting the Lohmann-cells generated by Subroutine BITS and (2) photographically reducing this 10 inch by 10 inch array to form the hologram. The physical size of the hologram is made to be consistent with the optical bench used in the reconstruction process. Collimators, irises, and lenses are used as required.

Main program HOLPLO is used to make the plot from which the hologram is formed. The photograph was made by illuminating the plot with two 100 watt flood lamps. The hologram was recorded on ortho-type film, exposure time one minute at $f\ 5.6$. The camera used was a Practina with standard lens. The exposure time and f -stop values were determined by trial and error. Since the end product is a binary hologram, film non-linearities do not present a problem.

For this report, the plotter outputs were chosen to have white backgrounds with opaque apertures to minimize noise introduced by the plotting equipment. Figures 16, 17, and 18 show Gould plots for three different clipping and bandlimiting values. Figure 16 is the plot for near-optimum parameter values as determined by OPT1. Figure 17 shows the plot set for the optimum number of frequency terms but with the amplitude clipped to 0.1 of the value of its maximum amplitude term. The plot of Figure 18 is for the case of no amplitude clipping but the number of frequency terms has been reduced to 18.

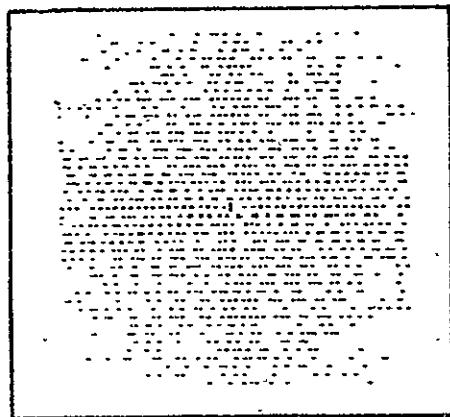


Figure 16. Gould Plot of Near-Optimum Binary Hologram Mask.

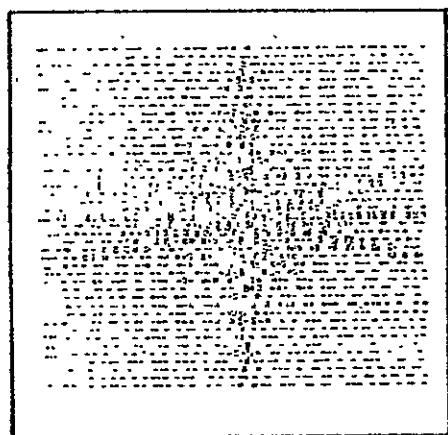


Figure 17. Gould Plot of Severely Clipped Binary Hologram Mask.

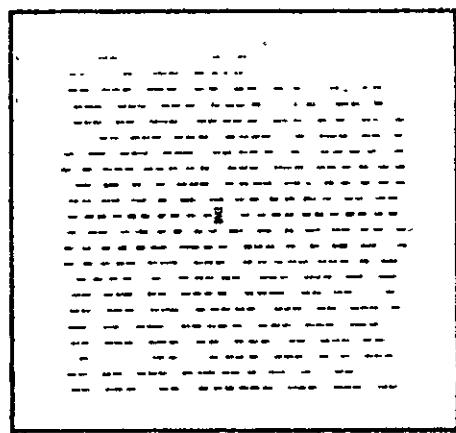


Figure 18. Gould Plot of Bandlimited Binary Hologram Mask.

The object used in making the previously described masks was the block letters AB. Optical reconstruction of the holograms produced for the three cases was accomplished by use of the optical arrangement shown in Figure 7. The reconstructed images were recorded on film located in the image plane. Figure 19 is the optical reconstruction with near-optimum parameter values. Figure 20 is the reconstruction with severe clipping of amplitude terms, while Figure 21 shows the bandlimited version of the reconstruction.

Optimum parameter values for synthesizing holograms are found by use of main program SN. This program allows both the bandwidth and amplitude clipping values to be varied in order that the signal to noise ratio of the resulting hologram may be observed. Table 1 shows signal to noise values of the hologram of AB as clipping and bandlimiting are varied. The clipping level is set by truncating any magnitude term greater than some preset percent of the maximum amplitude term of the transform array. The level is set by

$$\text{PCT} = \text{N}/9. , \quad \text{N} = 1, 2, \dots, 9. \quad (31)$$

The number of frequency terms NT is determined as in Subroutine BUILD by parameter NQ, where

$$\text{NQ} = 3*\text{M} , \quad \text{M} = 1, 2, \dots, 13. \quad (32)$$

The optimum clipping and bandlimiting parameters are determined by finding the maximum signal to noise figure from the chart of Table 1.

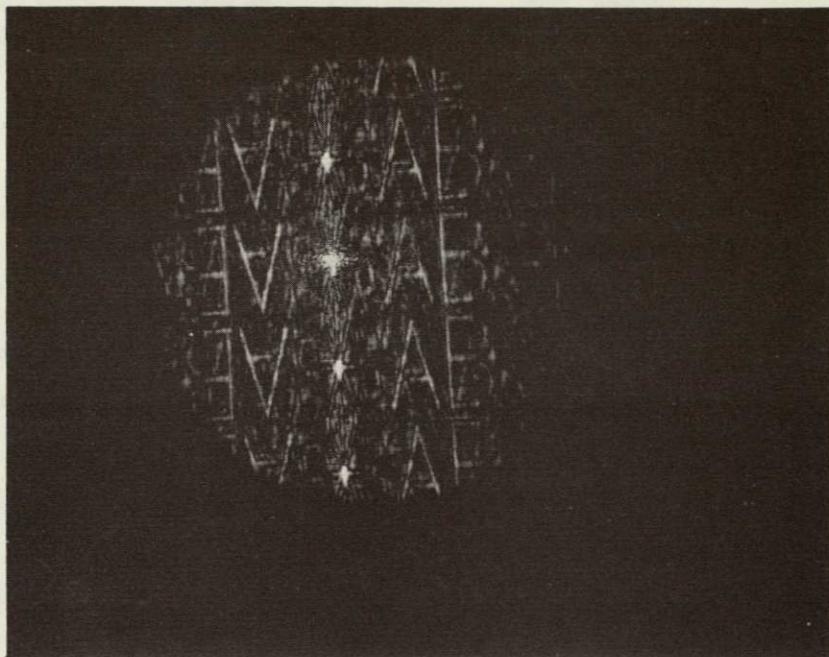
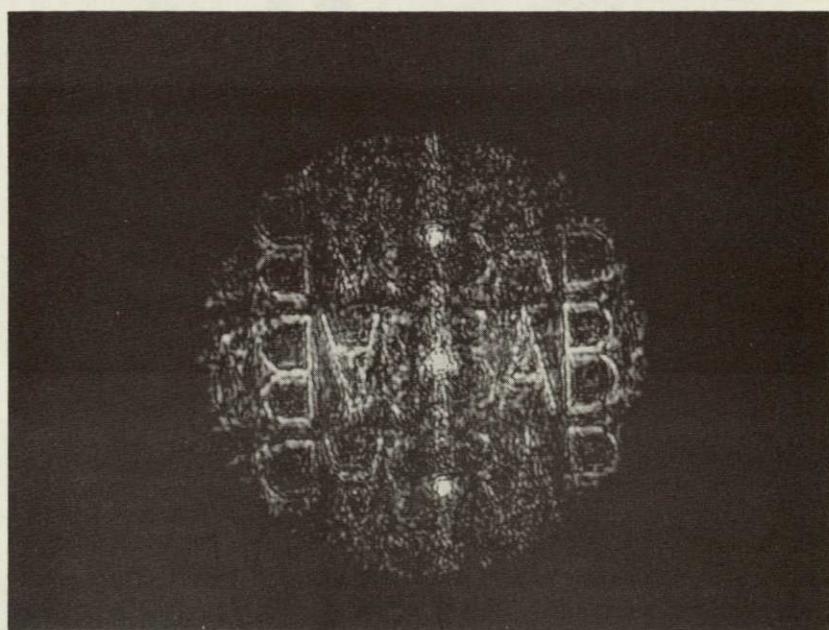


Figure 19. Optical Reconstruction of Near-Optimum Hologram.



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Figure 20. Optical Reconstruction of Hologram with Severe Clipping.

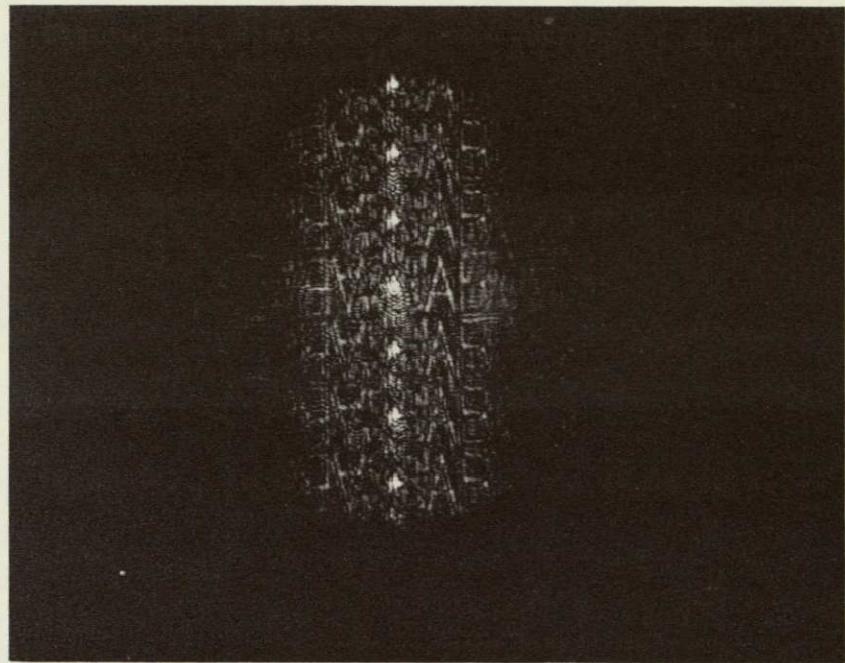


Figure 21. Optical Reconstruction of Bandlimited Hologram.

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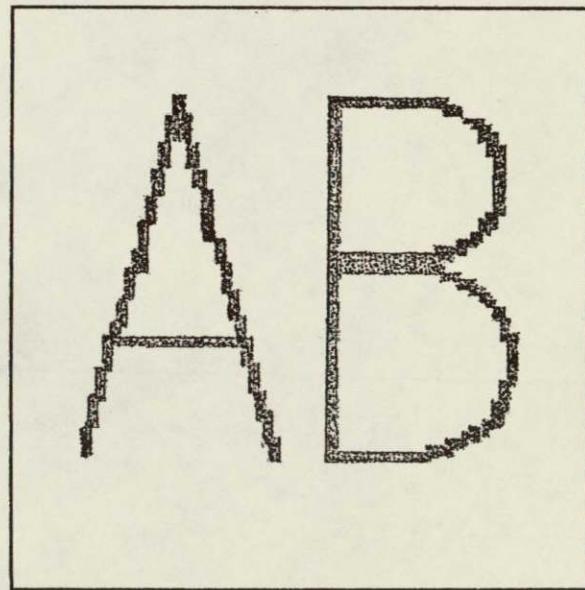


Figure 22. Simulated Reconstruction of AB without Degradation.

TABLE 1. Signal to Noise Values for the Hologram AB as Clipping and Bandlimiting Parameters are Varied.

PCT	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9	9/9
NQ									
3	1.23133	1.26116	1.30343	1.34114	1.37070	1.39255	1.40629	1.41311	1.41527
6	1.37075	1.42290	1.47815	1.52640	1.56493	1.59325	1.61160	1.62049	1.62085
9	1.53503	1.63277	1.70643	1.77311	1.82809	1.87095	1.90109	1.91725	1.92186
12	1.73836	1.88801	1.98742	2.07706	2.15273	2.20923	2.25094	2.27382	2.27968
15	1.96549	2.18778	2.31967	2.44189	2.54602	2.63231	2.68817	2.72577	2.73226
18	2.21838	2.51712	2.68914	2.85310	2.99298	3.10394	3.18864	3.22791	3.23327
21	2.49294	2.90470	3.13744	3.35534	3.54824	3.71513	3.80620	3.86246	3.85195
24	2.80401	3.34233	3.64422	3.93939	4.20581	4.40654	4.53874	4.57331	4.55456
27	3.16853	3.88491	4.28995	4.67748	5.03094	5.27615	5.38977	5.41485	5.31762
30	3.74277	4.82634	5.43430	6.00485	6.45327	6.73444	6.83529	6.70898	6.38879
33	4.27517	5.73963	6.48660	7.17552	7.54540	7.72156	7.55086	7.13220	6.68360
36	5.01664	7.03466	7.79711	8.18133	8.28672	7.97956	7.48325	6.86054	6.29099
39	5.42138	7.26897	7.27034	7.11410	6.68921	6.25740	5.82334	5.30017	4.83029

For the object employed in this study, the maximum signal to noise figure is 8.28672, as indicated in the table, and occurs when the clipping level is 5/9 of the maximum frequency domain number, and with NQ equal to 36. A value of 36 for NQ corresponds to 52 frequency terms, as determined by block data BLOCK and common QUAN.

A more efficient method of finding the optimum signal to noise value for a synthetic hologram and its associated clipping and bandlimiting parameters makes use of the program OPT1 and subroutine OPT. OPT1 simply reads in the binary mask from which the hologram is to be constructed. OPT is the non-linear optimization routine which performs the search for a maximum signal to noise value as the two parameters PCT and NQ are varied. It should be noted that the signal to noise figure is a non-linear function of the parameters, and additionally that the parameters are constrained to vary in discrete steps. A computer listing of the signal to noise figures and the corresponding parameters as the optimization routine searches for a maximum is given in Table 2. The starting point of the search routine is arbitrary; however, based on some knowledge of the function and the desired output, the solution is found faster by starting with a high NQ value. The printed outputs are signal to noise values found during the search, with the last line listing the maximum signal to noise figure along with the optimum parameter values.

TABLE 2. Computer Printout of Optimization
Program OPT.

NQ=39	CL= 3	S/N= 7.27034
NQ=36	CL= 3	S/N= 7.79711
I=11	J= 2	S/N= 5.73963
I=11	J= 3	S/N= 6.48660
I=11	J= 4	S/N= 7.17552
I=12	J= 2	S/N= 7.03466
I=12	J= 4	S/N= 8.18133
I=11	J= 3	S/N= 6.48660
I=11	J= 4	S/N= 7.17552
I=11	J= 5	S/N= 7.54540
I=12	J= 3	S/N= 7.79711
I=12	J= 5	S/N= 8.28672
I=11	J= 4	S/N= 7.17552
I=11	J= 5	S/N= 7.54540
I=11	J= 6	S/N= 7.72156
I=12	J= 4	S/N= 8.18133
I=12	J= 6	S/N= 7.97956
I=13	J= 4	S/N= 7.11410
I=13	J= 5	S/N= 6.68921
I=13	J= 6	S/N= 6.25740

THE OPTIMUM BANDWIDTH AND CLIPPING LEVEL VALUES ARE:

NQ	CLIP	F(12, 5)
36	.5556	8.28672

For comparison purposes, the three cases described above were reconstructed utilizing main program SIMUL. The first example simply illustrates the ability of the simulation routine to reconstruct an image without degradation. The uncorrupted reconstruction of AB is shown in Figure 22. The irregularities in the letters are due to quantization in the input binary mask. Figures 23, 24, and 25 are simulated reconstructions of optimum, severe clipping, and band-limiting cases, respectively.

Main program RECON is a simulation routine which recovers the hologram exactly as it was recorded by the plotter. Bandlimiting and clipping effects on hologram quality have been observed and studied in terms of a signal to noise ratio figure. RECON includes clipping and bandlimiting effects as well as phase and amplitude quantization introduced by the plotting equipment. Figure 26 shows a simulated reconstruction of AB after it has suffered plotter degradation. In addition, main program SNPLOT analyzes the signal to noise ratio of the degraded image; this will be discussed in the following chapter.

In order to test the simulated optical processing system, the famous Abbe-Porter wire mesh experiment was performed.¹⁵² Figure 27 shows the binary mask used to describe the mesh and Figure 28 is a horizontal slit filter. The results are as recorded by the simulator in Figure 29. If the slit is rotated by 90°, the results are as shown in Figure 30.

Further experiments with matched filters were performed utilizing the simulated optical processor. The character T as shown in Figure 31 was used to produce a matched filter as described in



Figure 23. Simulated Reconstruction, Optimum Clipping and Bandlimiting.

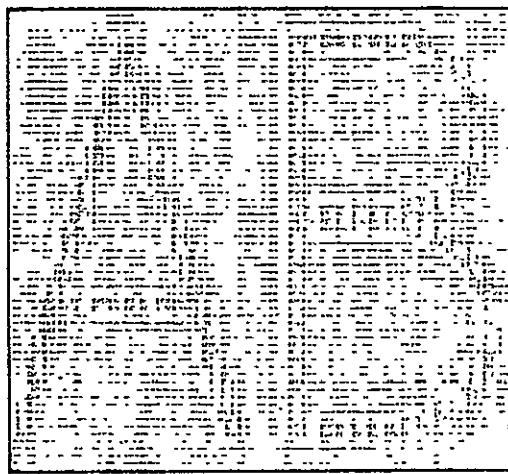


Figure 24. Simulated Reconstruction, Severe Clipping.

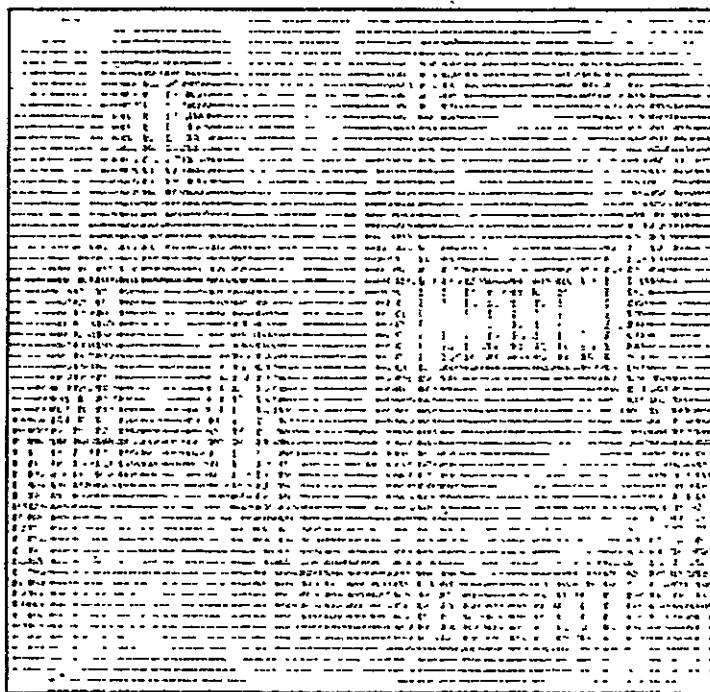
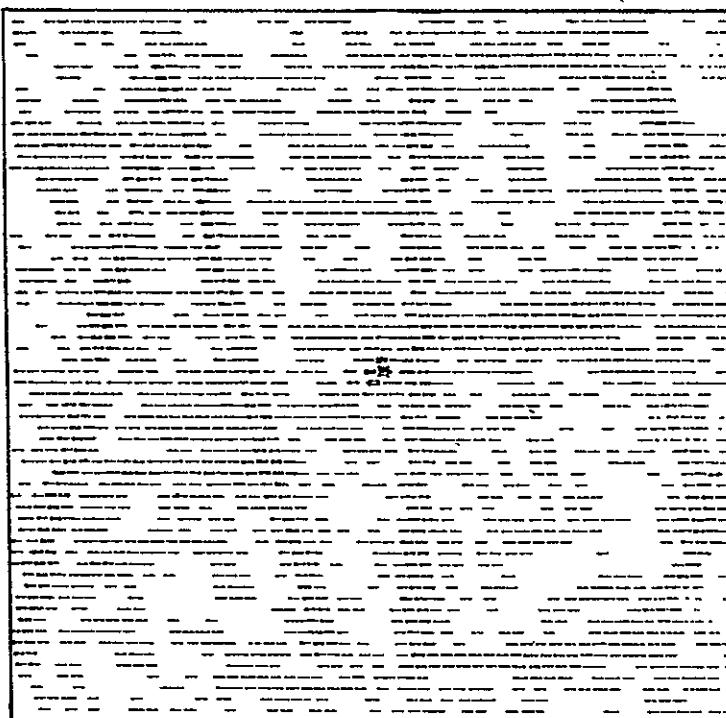


Figure 25. Simulated Reconstruction, Band-limited Version.



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Figure 26. Simulated Reconstruction Including Plotter
Induced Degradation.

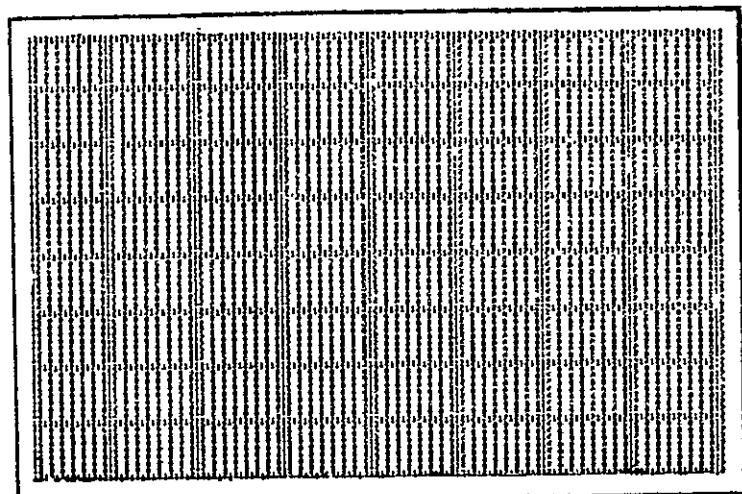


Figure 27. Binary Mask of Mesh for Abbe-Porter Experiment.

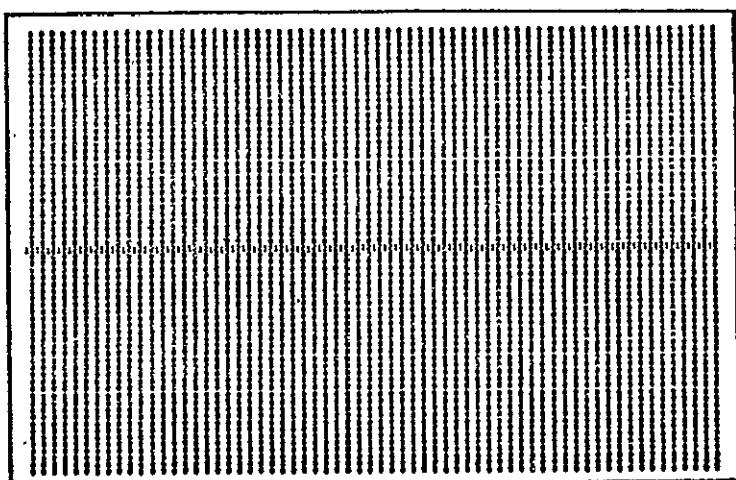


Figure 28. Horizontal Slit Filter.

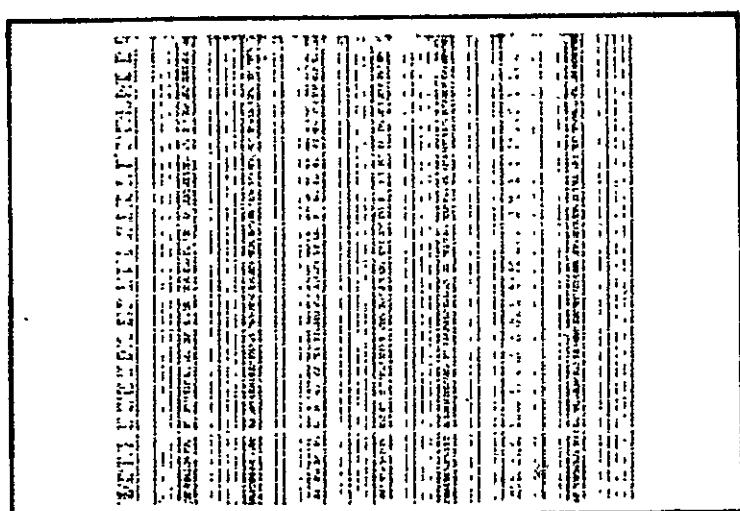


Figure 29. Simulated Reconstruction of Abbe-Porter Experiment with Horizontal Slit Filter.

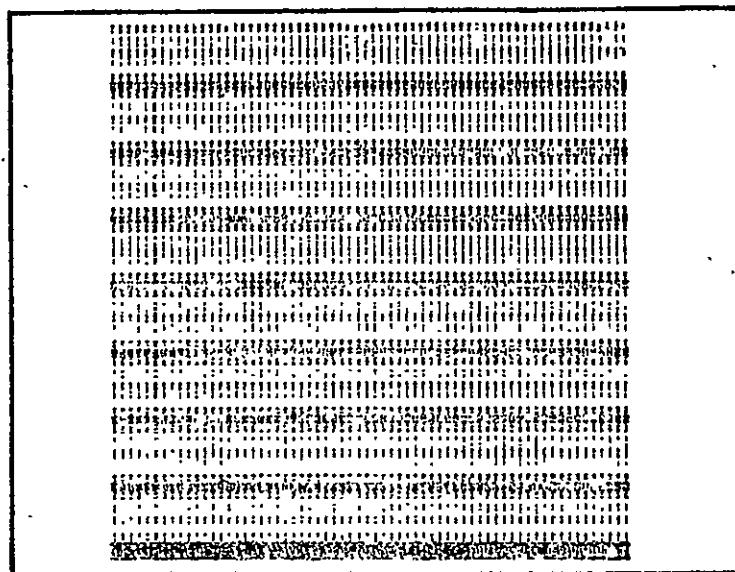


Figure 30. Simulated Reconstruction of Abbe-Porter Experiment with Vertical Slit Filter.

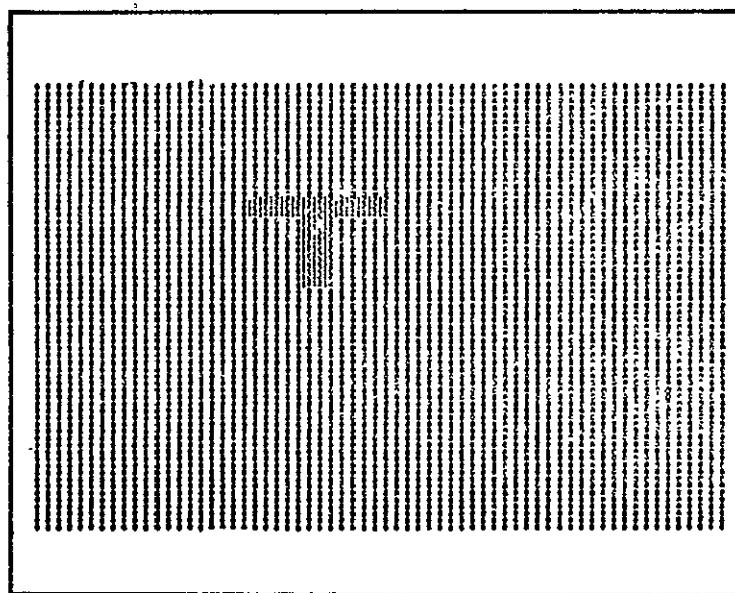


Figure 31. Character T used for Producing Matched Filter.

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Chapter I. The simulated autocorrelation of an input character T with the matched filter T is plotted in Figure 32. The optical autocorrelation of T with the matched filter T is photographed in Figure 33. Observe that in the optical reconstruction there is a reimaging of the input character and the convolution of the two characters which do not occur in the simulated field of reconstruction. Figure 34 is the result of the input character X which is not matched to the filter T . As in the previous example, Figure 35 is a photograph of the optical autocorrelation of the unmatched filter and input. To further illustrate that the simulator functions as an optical processor, Figure 36 presents results of changing the position of the character T in the input aperture.

The final experiment with matched filtering was with character recognition. Once again, the character T of Figure 31 was used as the filter. The mask OXT of Figure 37 was applied as the input. The simulated autocorrelation of the input characters OXT with the filter T is plotted in Figure 38. The darker plot on the right side of the figure aligns with the input symbol T and represents the autocorrelation of this input with the filter T . The center spot is the cross correlation of the input X with the filter T , and the left-most spot indicates the crosscorrelation of the input O with the filter T . An optical autocorrelation of the same character recognition system is photographed in Figure 39.

The last effort in the study deals with encoding and decoding holograms. The characters AB are used as the message symbols. Object description AB is Fourier transformed by HOLPLO in the usual

manner. After the transformation, a reference symbol, which was chosen to be 0 for the study, is also Fourier transformed. A hologram is then produced of $F[AB]$ divided by $F[0]$. The result is an encoded hologram $F[AB]/F[0]$ which is plotted and photographed as previously described. Theoretically, the original information symbols are recoverable by optically multiplying $F[0]$ by the encoded hologram, although the optical reimaging of the encoded symbols was not successful in this study. Unfortunately, recovery of the encoded information is extremely difficult due to critical alignment of the synthetic holograms. Recovery of the encoded symbols is possible, however, by use of the simulation routine and the results are plotted in Figure 40.

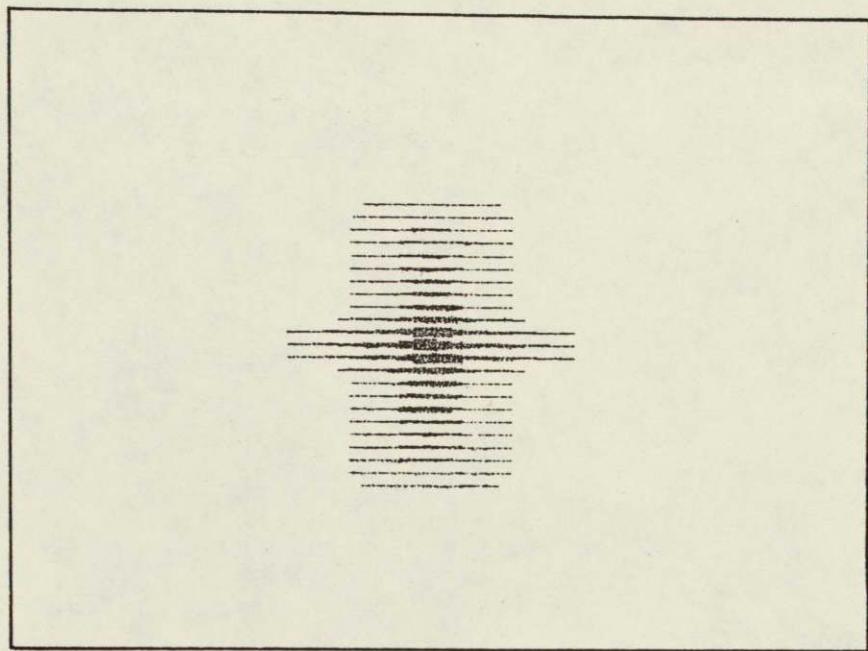


Figure 32. Simulated Autocorrelation of Input Character
 T with Matched Filter T .

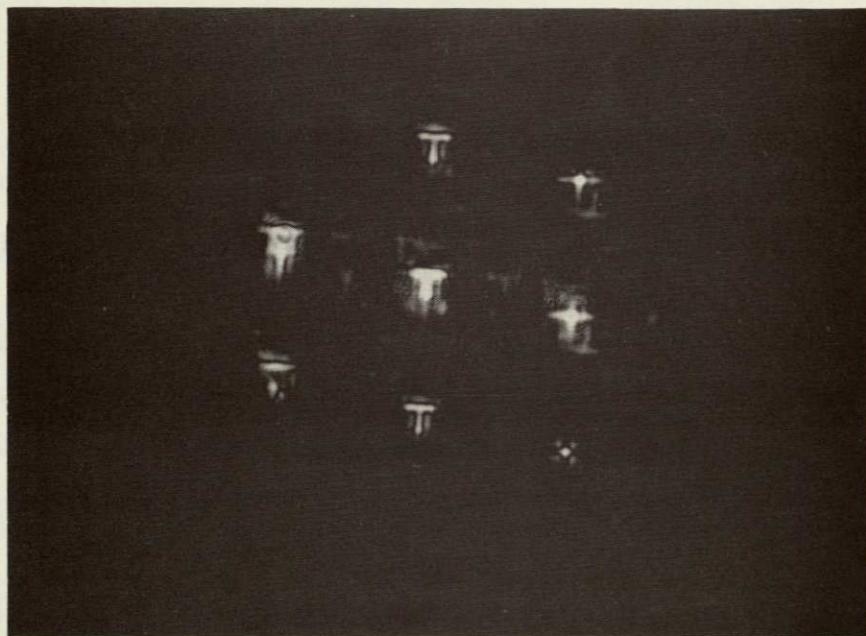


Figure 33. Optical Autocorrelation of Input Character
 T with Matched Filter T .

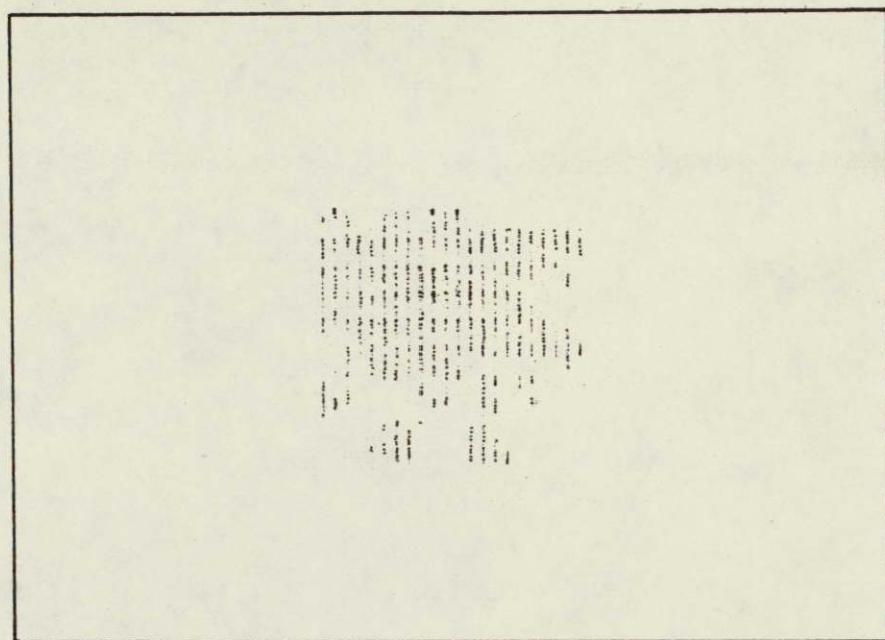


Figure 34. Simulated Output of Optical Processor with Input Character and Filter Unmatched.

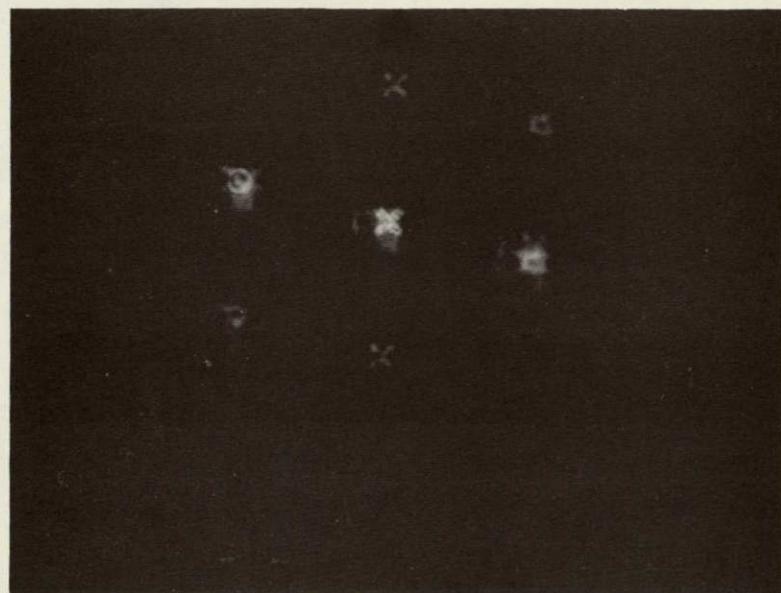


Figure 35. Optical Reconstruction of Autocorrelation with Unmatched Filter.

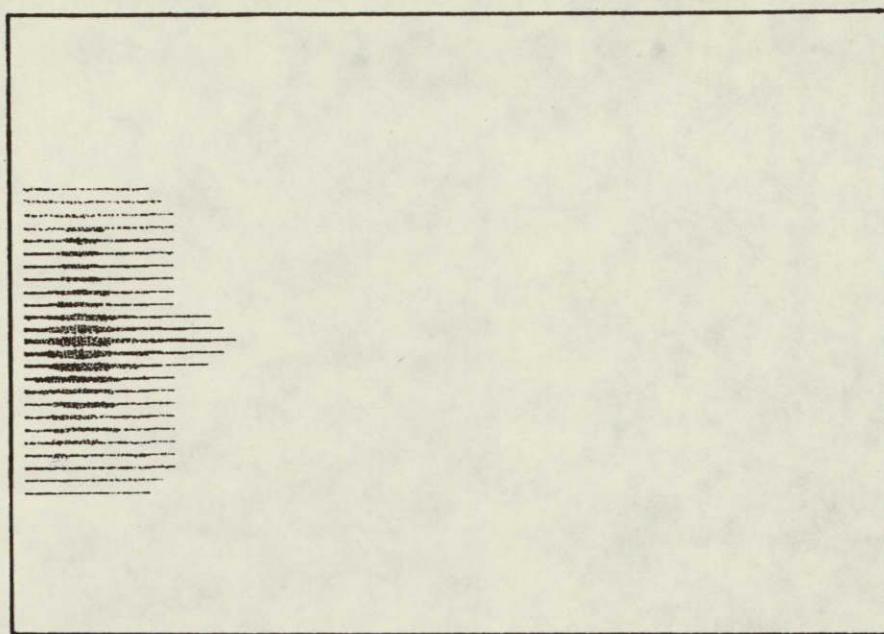


Figure 36. Simulated Autocorrelation of Input Character T with Aperture Position Shifted.

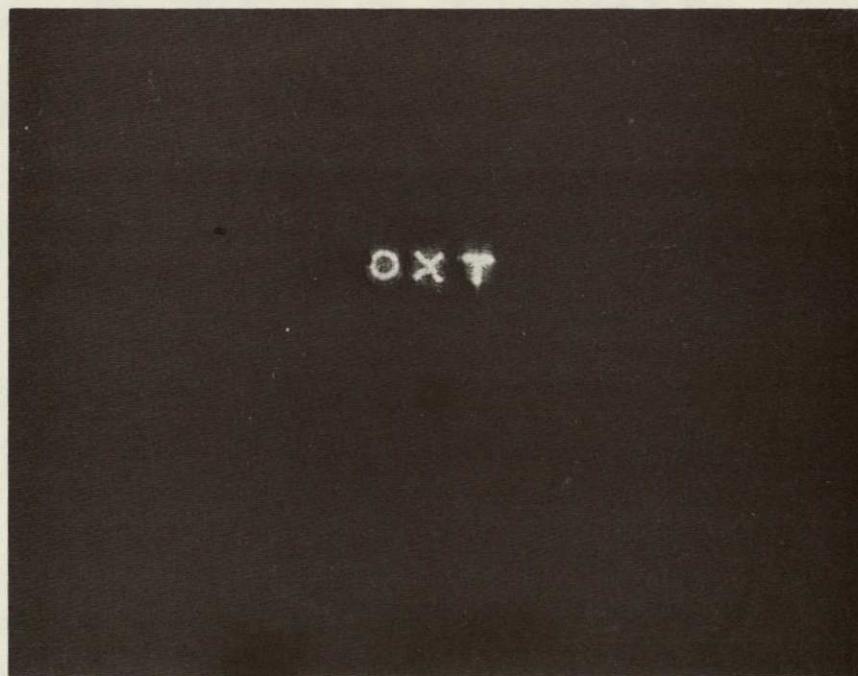


Figure 37. Input Mask OXT for Character Recognition Study.

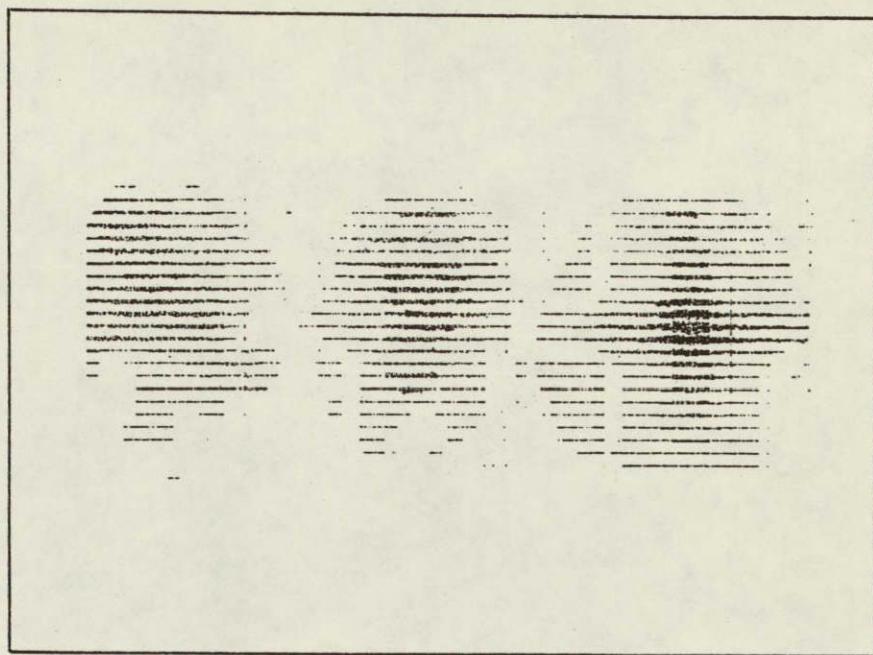


Figure 38. Simulated Autocorrelation of Input OXT
with Matched Filter T .

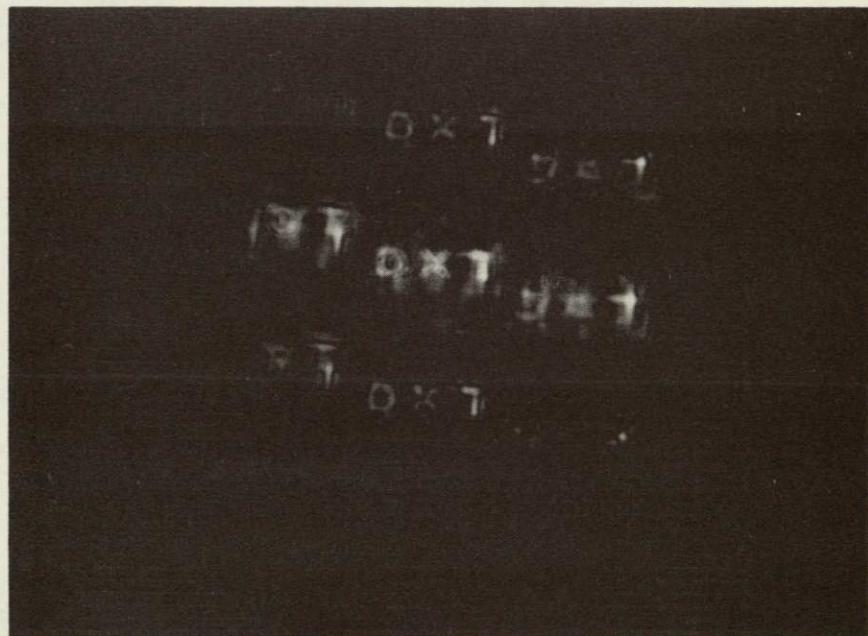


Figure 39. Optical Autocorrelation of Input OXT
with Matched Filter T .

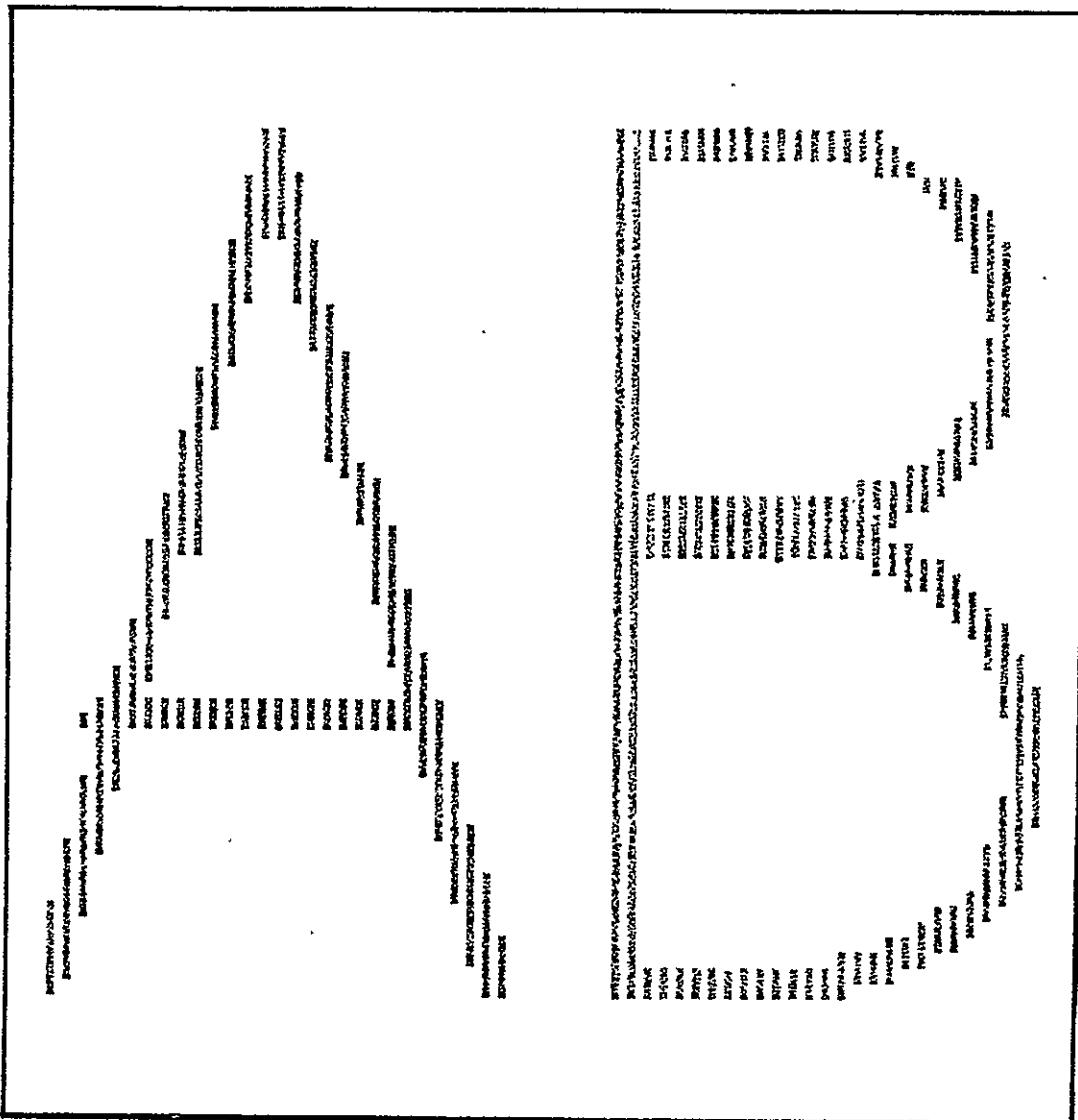


Figure 40. Simulated Réconstruction of the Hologram
 $F[AB]/F[0]$.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The results of this research are oriented more toward applications than to pure theory. Most of the conclusions drawn from the experiments are based on physical results. Of course, the mathematics of Fourier optics is the basis for the synthesis of Lohmann-type holograms. Also, techniques used for finding optimum signal to noise values are founded on mathematics.

The synthetic holograms produced in this study reconstruct images as predicted by the theory of optical holography. The Gould plotted binary masks yield a hologram which produces a high quality image reconstruction. The optimization algorithm provides a method for determining parameters for synthesizing near-optimum holograms.

Photographs of optical reconstructions reveal that the selected parameters produce holograms of quality as predicted. Figure 19 is an optical reconstruction of high quality as selected by the optimization routine.

Selection of clipping and bandlimiting parameters may be made by varying those quantities and observing their effect on the resulting signal to noise ratio. Typical computer run time for the program SN to tabulate signal to noise ratios is in the order of 8 minutes. Application of non-linear programming techniques as in Subroutine OPT reduces computer time to approximately 3 minutes and directly provides

parameter values to the hologram plotting routine. Thus, program OPT saves both operator time and computer run time.

Optical simulator plots reveal the effects of clipping and band-limiting variations on image reconstruction. The simulator allows image reconstruction to be observed without the necessity of using a laser and optical bench. Note the high quality of reconstruction in Figure 23 with parameters set for optimum values. The simulated reconstruction of Figure 24 displays differentiated edges due to the clipping value selected. Finally, in Figure 25, the absence of high frequency terms due to bandlimiting causes a rounding of the leading edges of the image. For purposes of reconstructing images, the optical simulation routine provides reconstructions which agree precisely with optical reconstructions.

Program RECON was devised to reconstruct the image from the hologram exactly as it was recorded by the plotter. The unique feature of RECON is that it takes into account both phase and amplitude quantization as introduced by the plotting equipment. The optimization program finds an optimum hologram in terms of clipping and bandlimiting only; thus, the important contribution of plotter induced errors has been disregarded in the making of the hologram. The plot shown in Figure 26 is a simulated reconstruction as recovered from the plot tape. The parameters for this hologram were the same as used in making the optimum holograms of Figures 19 and 23. Obviously, there is more degradation than was present in the reconstruction through the simple optical simulator.

The additional degradation of the image reconstructed from the plotter is due primarily to phase and magnitude quantization as discussed in Chapter II. In addition to quantization errors, there is some loss of magnitude and phase information due to occasional overlapping of apertures in adjacent cells. A signal to noise study was run on the hologram recovered from the plotter. Main program PLOTSN was used. For the clipping and bandlimiting parameters selected, the signal to noise ratio of the hologram prior to plotting was 7.5454. After plotter degradation, the signal to noise ratio dropped to 3.3606, a decrease of image intensity to 48% of its original value.

For testing the spatial filtering capability of the simulated optical processor, the Abbe-Porter mesh was applied as an input and a horizontal slit was used as a filter as shown in Figures 27 and 28, respectively. The result was a complete suppression of the horizontal components of the mesh as verified by Figure 29. Rotating the slit filter by 90° suppresses the vertical components of the mesh in the reconstruction as seen in Figure 30.

One of the outstanding features of computer generated holograms is their use as spatial filters. Once a method for generating synthetic holograms has been devised, spatial filters for character recognition and autocorrelation studies may be synthesized with ease. The real advantage of synthetic binary spatial filter generation is that the necessity of using an optical bench, liquid gates, and recording apparati is alleviated.

The program HOLPLO was used for generating synthetic binary spatial filters and the simulation routine SPAFIL was used for

simulating the results of the spatial filtering experiments. The experiments involved autocorrelation and character recognition studies.

In the spatial filtering experiments, the symbol T was used as the filter. Various input characters were applied and the results were observed by both the simulator plots and the optical reconstructions. Figure 32 is a plot of the autocorrelation of the input symbol T with the matched filter T . Note that only the autocorrelation from the simulator is plotted. For comparative purposes, a synthetic binary filter of the symbol T was made utilizing HOLPLO. With a mask T as the input to an optical processor as shown in Figure 2, the optical reconstruction is photographed in Figure 33. In the photograph, the autocorrelation spot appears to the right, the input T is reimaged near the center, and the convolution spot appears on the left side of photograph. Figures 34 and 35 show the results of an input character X which is not matched to the filter T . Note that there is some cross correlation, but not to the degree observed in the matched case. One further verification of the versatility of the optical simulator was shown by shifting the position of the symbol T in the input aperture and observing the corresponding shift of the autocorrelation spot in Figure 36.

A character recognition device was simulated by applying the input characters OXT to the optical processor. The simulated autocorrelation with the filter T is plotted in Figure 38. Notice that the autocorrelation spot aligns with the character T of the input binary mask. A mask of the characters OXT was fabricated and used as the

optical input to the system of Figure 2 using the synthetic binary filter \bar{T} . The optical results are photographed in Figure 39. The results obtained using the synthetic filter appear to be of quality comparable to those systems employing the Vander Lugt filters.¹⁵²

One certain conclusion that can be drawn from the hologram coding study is that optical decoding of synthetic holograms is extremely difficult. Subroutine OPT2 and a modified HOLPLO program were employed to find the optimum parameters and synthesize a binary hologram of the space domain division of two Fourier transform arrays. Successful results were achieved employing the simulation routine as shown in Figure 40; however, optical recovery of the encoded symbol was never attained. The presence of the information in the encoded hologram was verified by autocorrelation, but due to optical alignment problems, a legible reconstruction was unsuccessful.

There are two contributions of this study which are significant. First, the optimization routine allows hologram parameters to be selected which produce the best possible computer generated hologram. The fact that the hologram is optimal has been verified by both optical simulation and by the laser and optical bench. The second contribution is the simulated optical processor which performed in the same manner as a classical optical processor. The simulation of image reconstruction allowed experimental results to be observed without the use of lasers and optical benches. The outstanding result obtained from the simulator was the image reconstruction directly from the plotter tape. Access to the plotted hologram allowed degradation

effects of phase and amplitude quantization to be observed as well as the standard computer constraints.

Recommendations

Based on the results of this research, it may be concluded that Lohmann-type holograms are feasible for use in data acquisition, storage, and transmission. There is definitely degradation of input information during the process of constructing the synthetic hologram, but in light of the relatively small amount of computer storage required to represent a large amount of data, some degradation is acceptable. Future studies should consider the use of Lee-type holograms and gray-scale plotters with an eye toward minimizing quantization errors.

A very promising area of application of computer-generated holograms is that of synthetic-aperture radar. Optical information processing techniques have been employed successfully in high-resolution synthetic aperture radar systems. Further investigations should include synthetic hologram recordings for these mappings.

During the course of this study an attempt was made to observe contrast reversal effects in image reconstruction by filtering the zeroth order term of the Fourier transform hologram. This idea could be pursued with some possible applications in microscopy and image enhancement.

The results of the correlation and character recognition studies were as predicted by theory and verified by plots and photographs. Possible extensions of this phase of the study should include an

intensity measurement of the optical reconstructions and a computer algorithm for comparing relative intensities of the simulated correlations.

Finally, the computer routine for optimizing the synthetic holograms performed exceedingly well and produced optical and simulated results as expected. In the reconstruction routine RECON, there is a significant degradation of the reconstructed image due to plotter quantization effects. Therefore, it is recommended that a program be developed for determining optimum parameter values with plotter induced degradations being taken into account.

APPENDICES

APPENDIX A

BIBLIOGRAPHY

1. Literature Survey

In an effort to appraise the state of development of computer generated holograms and spatial filters, an extensive survey of holographic related literature was undertaken. The dates of the survey extend from the beginning of computer holography in the mid-1960's to the present time. The majority of this literature was available in the library of the Mississippi State University. A bibliography of computer generated holography and related topics has been compiled and appears in Appendix A. The sources of the articles listed and the inclusive search dates are as follows:

1. Applied Optics, January 1967 through April 1976.
2. Applied Physics Letters, January 1966 through May 1976.
3. Dissertation Abstracts International, B, The Sciences and Engineering, January 1967 through April 1976.
4. IBM Journal of Research and Development, January 1967 through March 1976.
5. Institute of Electrical and Electronics Engineers Proceedings, January 1967 through December 1975.
6. Institute of Electrical and Electronics Engineers Spectrum, January 1967 through May 1976.
7. Institute of Electrical and Electronics Engineers Transactions on Computers, January 1967 through April 1976.

8. Journal of the Optical Society of America, January 1967 through April 1976.
9. Laser Abstracts, January 1972 through May 14, 1976.
10. Laser Focus, January 1967 through May 1976.
11. Nature, January 1967 through February 1976.
12. Optica Acta, January 1967 through March 1976.
13. Optical and Quantum Electronics, January 1975 through May 1976.
14. Optical Engineering, January 1972 through February 1976.
15. Optical Scanning News, January 1974 through May 1976.
16. Optics and Spectroscopy, January 1967 through November 1975.
17. Optics Communications, January 1969 through April 1976.
18. Opto-Electronics, January 1969 through December 1974.
19. Physics Letters, April 1967 through February 1976.
20. Science, January 1967 through March 1976.
21. SPIE Journal, January 1971 through December 1972.

For the purpose of making a meaningful summary, the aforementioned topics will be broken into three categories: (1) computer generated holograms, (2) spatial filters and image processing, and (3) applications of computer holography. Included in the summary of each category will be a list of the most pertinent articles with a synopsis.

Computer Generated Holograms

Several methods are presently in vogue for synthesizing holograms by means of a digital computer. Reference to a synthetic hologram

implies that a mathematical description has been used for forming an image and that a physical object need not be used. Of primary interest in this study is the binary hologram which has transmittance of either "1" or "0".

The earliest references to computer-generated holograms date back to the mid-1960's when Lohmann and Paris experimented with binary spatial filters. The binary Fraunhofer hologram synthesized by a digital computer was proposed in a paper by Lohmann and Paris.¹² The hologram referred to in this article is known as the Lohmann-type hologram. This type of hologram subsequently has been improved in terms of reconstruction quality.³² Studies also have been made of reconstruction errors due to quantization in Lohmann holograms.⁵¹

Two and three-dimensional holograms as well as color holograms have been synthesized by the digital computer. One of the recent synthetic holograms is known as the Lee-type and employs the positions of the samples of the synthesized hologram to record phase information of a complex wavefront.⁴⁹ The kinoform is a computer generated wave-front reconstruction device which operates only on the phase of an incident wave.³³ More recently, a class of holograms called circular-carrier holograms (CCH) has appeared.¹³⁷

Interest in computer generated holography appears to have revived recently as evidenced by the number of articles and Ph.D. dissertations which have appeared during the last year. The current emphasis appears to be on applications of computer generated holograms.

Following is a chronological list of some of the most pertinent articles related to computer generated holography. A brief synopsis is included with each article in the list.

1. Lohmann, A. W. and D. P. Paris, "Binary Fraunhofer Hologram, Generated by Computer," Applied Optics, Vol. 6, No. 10, 1967.

A method for synthesizing holograms from mathematical descriptions is described. A computer plots a drawing of the hologram which in turn is reduced photographically.

Theory and experimental results are presented.

2. Lesem, L. B., P. M. Hirsch, and J. A. Jordan, Jr., "The Kinoform: A New Wavefront Reconstruction Device," IBM Journal of Research and Development, Vol. 13, No. 2, 1969.

A method for constructing an optical element which operates only on the phase of an incident wave is described. The kinoform exhibits high efficiency in terms of spatial frequency potential and reconstruction energy. Computer synthesis time is less than that for a digital hologram.

3. Brown, B. and A. W. Lohmann, "Computer Generated Binary Holograms," IBM Journal of Research and Development, Vol. 13, No. 2, 1969.

Improvements over the original Lohmann-type binary hologram are discussed. Some simplification in computer production and reconstruction of two and three-dimensional images are presented.

4. Lee, W. H., "Sampled Fourier Transform Hologram Generated by Computer," Applied Optics, Vol. 9, No. 3, 1970.

This paper describes a technique for determining a real non-negative function for representing the transmittance of a computer synthesized hologram. Positions of the samples in

the synthesized hologram record the phase information of a complex wavefront. This paper presents the Lee-type hologram.

5. Gabel, R. A. and B. Lin, "Minimization of Reconstruction errors with Computer Generated Binary Holograms," Applied Optics, Vol. 9, No. 5, 1970.

Degradation of image reconstruction in computer generated holograms due to equipment limitations is discussed. Truncation and quantization errors are studied. Theoretical and experimental results are presented.

6. Huang, T. S., "Digital Holography," Proceedings of the IEEE, Vol. 59, No. 9, 1971.

This paper summarizes techniques for generating and reconstructing computer-generated holograms. Lohmann's and Lee's methods of synthesizing holograms are included. Some applications of computer holography are presented.

7. Chu, D. C. and J. R. Fienup, "Recent Approaches to Computer-Generated Holograms," Optical Engineering, Vol. 13, No. 3, 1974.

A recent approach to computer synthesis of holograms and the production procedure is described. The method presented has the advantage of the kinoform without its limitations. Experimental results are included and the referenceless on-axis complex hologram (ROACH) is introduced.

8. Lee, W. H., "Binary Synthetic Holograms," Applied Optics, Vol. 13, No. 7, 1974.

This paper presents a method for making binary synthetic holograms of wavefronts with constant amplitude.

9. Lee, W. H., "Circular Carrier Holograms," Journal of the Optical Society of America, Vol. 65, No. 5, 1975.

Herein is presented a description of a computer generated hologram that is made with a reference wave having a linear phase variation in the radial direction. A method for making the circular-carrier hologram (CCH) and reconstructing the object wave front using a circular grating is formulated.

10. Hugonin, J. P. and P. Chavel, "A Complement to the Theory of Lohmann-type Computer Holograms," Optics Communications, Vol. 16, No. 3, 1976.

Brown and Lohmann improved their original holograms by using true phase and amplitude coding at the center of the diffracting aperture. This article presents a mathematical analysis of this procedure.

In addition to the ten articles listed above, the following references to the bibliography in Appendix A are also related to computer generated hologram: 1, 3, 6, 11, 12, 15, 18, 19, 20, 23, 24, 25, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 45, 48, 49, 50, 51, 52, 55, 56, 58, 60, 62, 64, 65, 66, 68, 69, 70, 71, 72, 74, 75, 76, 77, 79, 81, 82, 83, 86, 87, 89, 90, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 105, 106, 107, 109, 110, 111, 113, 116, 117, 118, 119, 120, 121, 122, 124, 127, 130, 131, 133, 134, 135, 137, 140, 142, 143, 145, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159.

Spatial Filters and Optical Image Processing

Computer-generated holograms are particularly well suited to spatial filtering applications because of their relative ease of

fabrication. In fact, filter functions which are difficult, if not impossible, to produce by classical optical methods may be synthesized by digital computer methods. Furthermore, as is the case with computer-generated holograms, no physical object is required for synthesizing the filter.

The development of computer-generated spatial filters closely parallels that of the computer-generated holograms. In 1966 Brown and Lohmann described a method for generating spatial filters by means of a digital computer. Their technique yielded a binary matched filter which exhibited the same properties as an optically fabricated filter.

In 1967 Burch introduced a computer algorithm for synthesizing computer-generated holograms to be used as spatial filters.⁷ Lohmann and Paris proposed the use of computer created binary spatial filters for applications in coherent optical data processing systems. Included in their optical processing applications were phase contrast demonstrations and gradient correlation filters. Another application of the binary spatial filter was its use as an inverse filter in an image restoration system.

Several variations of Lohmann's original filter have appeared in recent years. One of these is the kinoform applied to an incoherent optical processing system.⁵⁹ The possibility of optical processing with incoherent light increases the scope of applications of computer generated spatial filters. Incoherent optical-image processing with synthetic holograms and the resulting signal-to-noise ratios has been studied in some detail.¹⁴⁶

Computer-generated spatial filters may be synthesized in varying levels or shades of intensity as required for kinoform generation. The use of a multiple gray-level plotting device for recording synthetic holograms has been reported by Campbell, Wecksung, and Mansfield.¹¹⁵ A halftone plotter has been used for image restoration in a computer simulated optical processing system.⁴⁴

Following is a list of articles pertaining to computer-generated spatial filters and optical processing systems:

1. Brown, B. and A. W. Lohmann, "Complex Spatial Filtering with Binary Masks," Applied Optics, Vol. 5, No. 6, 1966.

This is one of the original papers on computer-generated holograms. A discussion of image restoration using binary holograms is included. Also of interest is a description of the use of a computer-generated hologram as a matched spatial filter.

2. Burch, J. J., "A Computer Algorithm for the Synthesis of Spatial Frequency Filters," Proceedings of the IEEE, Vol. 55, No. 4, 1967.

This letter describes a digital computer algorithm for synthesizing Fourier transform holograms to be used as spatial filters. Experimental results of filters made by this method are discussed. The Burch-type hologram is introduced.

3. Lohmann, A. W. and D. P. Paris, "Computer-Generated Spatial Filters for Coherent Optical Data Processing," Applied Optics, Vol. 7, No. 4, 1968.

In this paper the authors discuss binary spatial filters which are similar to Lohmann-type holograms. It is shown

that these filters can perform any optical processing operation producible by an optical filter; experimental results are included.

4. Weider, H., R. V. Pole, and P. F. Heidrick, "Electron Beam Writing of Spatial Filters," IBM Journal of Research and Development, March 1969.

A method for generating spatial filters by use of electron beam-addressed KDP crystals is described in this article. Some examples of simple filters made by this method are presented.

5. Ichioka, Y., M. Izumi, and T. Suzuki, "Halftone Plotter and its Applications to Digital Optical Information Processing," Applied Optics, Vol. 8, No. 12, 1969.

A digital computer is used to simulate an optical processing system. Examples of spatial filtering and image restoration utilizing computer holography are given.

6. Patau, J. C., L. B. Lesem, P. M. Hirsch, and J. A. Jordan, "Incoherent Filtering Using Kinoforms," IBM Journal of Research and Development, Vol. 14, No. 5, 1970.

Application of the kinoform as a filtering element is described in this article, and its advantage over the Lohmann type hologram is discussed. Some results of correlation filters are presented.

7. Ransom, P. L. and R. M. Singleton, "Synthetic Real-Imaginary Hologram Pair," Applied Optics, Vol. 13, No. 4, 1974.

Two holograms, one representing the real part of the complex wavefront, the other representing the imaginary part, are synthesized. The computer generated hologram pair is illuminated and the resulting reconstructions are added in phase quadrature to produce the desired complex wavefront.

8. Campbell, K., G. Weksung, and C. Mansfield, "Spatial Filtering by Digital Holography," Optical Engineering, Vol. 13, No. 3, 1974.

A multiple gray-level plotting device for constructing digital holographic spatial filters is presented. Examples of the synthesized filters in a coherent optical processing system are shown. The theory of sampled holograms and spatial filters is discussed in some detail.

9. Lin, B. and N. C. Gallagher, "Optimum Fourier-Transform Division Filters with Magnitude Constraint," Journal of the Optical Society of America. Vol. 64, No. 9, 1974.

This paper includes a discussion of Fourier transforms. The optimum spatial filter is developed using integrated squared error as a fidelity criterion with the magnitude of the filter transfer function subject to a constraint. Primary considerations in this article are given to image transformation and various methods of improving image reconstruction.

10. Chavel, P. and S. Lowenthal, "A Method of Incoherent Optical Image Processing Using Synthetic Holograms," Journal of the Optical Society of America, Vol. 66, No. 1, 1976.

A Comparison of signal to noise in an incoherent optical processing system to that under coherent illumination is made. Image deblurring is discussed, and image processing employing incoherent illumination on computer-generated holograms of a low number of cells is presented.

Other papers, articles, letters, or books which are related to spatial filtering and/or optical image processing are listed in the

bibliography of Appendix A. The following list of numbers indicate those entries in the bibliography: 1, 2, 4, 7, 9, 10, 14, 16, 20, 22, 27, 43, 44, 45, 46, 50, 53, 59, 61, 73, 75, 81, 84, 98, 103, 104, 106, 107, 110, 111, 115, 119, 129, 131, 132, 133, 136, 146, 152, 154, 156, 165.

Applications of Computer-Generated Holography

The appeal of the computer-generated hologram is its construction from a mathematical description of any object--one, two, or three-dimensional. There are numerous applications of these easily fabricated devices. Among the list of applications are mass data storage, image manipulation, and optical element testing.

An early application of holography was proposed by Gabor in the area of character recognition.² Later, this idea was extended to include Lohmann-type holograms and the recovery of coded information.⁸ Basically, these techniques are applications of the spatial filters discussed earlier in this paper.

A promising utilization of computer-generated holograms is mass data storage of digital computers.⁵⁴ Holographic storage offers the advantage of rapid accessibility and low cost compared with presently used mass storage media.

Spatial filters generated by the computer have been used in optical image processing systems. Holography has been applied to the focusing of blurred images and to the enhancement of images degraded by instrument errors or other causes.^{29,40} This area of application is related to matched filtering and has been used with

both coherent and incoherent illumination. Other useful areas in computer holography include the reproduction of three-dimensional objects and color images.

A very practical use of computer-generated holograms has been the testing of optical surfaces. Several papers described aspheric lens testing and applications of computer holography to curved surfaces.^{67,88,128,150} A related area of application is shearing interferometry.¹²³ In addition to optical testing, computer-holograms have been used for producing kinoform lenses, zone plates, and diffraction gratings.⁵⁷

Finally, a number of holographic apparatus have been developed and patented. These devices include holographic recorders and character recognition machines. See U.S. Patent descriptions listed in Appendix A. The following list of articles applies to applications of computer-generated holograms:

1. Gabor, D., "Character Recognition by Holography," Nature, Vol. 208, Oct. 30, 1965.

A utilization of holography in the recognition of characters with many variants is presented in this article by Dennis Gabor, one of the pioneers in holography. It is proposed that holograms have properties which can discriminate between numerals and letters of the alphabet.

2. Lohmann, A. W., D. P. Paris, and H. W. Werlich, "A Computer-Generated Spatial Filter, Applied to Code Translation," Applied Optics, Vol. 6, No. 6, 1967.

An extension of Gabor's holographic character recognition system is proposed which utilizes computer-generated

holograms. Some of the problems of producing the spatial filters are reported and experimental results are shown.

3. Stroke, G. W. and R. G. Zech, "A Posteriori Image-Correcting 'Deconvolution' by Holographic Fourier-Transform Division," Physics Letters, Vol. 25A, No. 2, 31 July 1967.

Sharpening of defocused images and correction of instrument induced faults in photographic recordings are shown to be correctable by holographic methods. Although techniques formulated in this letter refer to classical holography they are adaptable to computer generated filters.

4. Hickling, R., "Scattering of Light by Spherical Liquid Droplets Using Computer Synthesized Holograms," Journal of the Optical Society of America, Vol. 58, No. 4, 1968.

A proposal for determining diameters of spherical liquid droplets in the range of 0.5 to 20 microns is presented. The method is based on techniques used in the construction of holograms synthesized from far field illumination scattered by the droplets.

5. Jordan, J. A., P. M. Hirsch, L. B. Lesem, and D.L. Van Rooy, "Kinoform Lenses," Applied Optics, Vol. 9, No. 8, 1970.

The production of accurate, thin, and light weight lenses is discussed in this paper. Large lenses which are very thin and light-weight compared with conventional lenses are constructed by kinoform techniques.

6. Fillmore, G. L., "Kinoform Viewed as a Random Number Generator," Applied Optics, Vol. 11, No. 10, 1972.

Kinoforms are shown to produce output intensities that are a random selection from some universal population.

7. Engel, A. and G. Herzinger, "Computer Drawn Modulated Zone Plates," Applied Optics, Vol. 12, No. 3, 1973.

A method of producing modulated zone plates by means of a digital computer is described. The modulated zone plates are compared to synthetic binary holograms.

8. Lee, W. H., "Contour Map Display Using Computer-Generated Holograms," Applied Optics, Vol. 14, No. 10, 1975.

This paper presents an application of Lee-type holograms in displaying contour maps of two-dimensional functions.

Interesting maps of the force-lines of an electric dipole are shown.

9. Takahaski, T., K. Kono, M. Kawai, and M. Isshiki, "Computer-Generated Holograms for Testing Aspheric Lenses," Applied Optics, Vol. 15, No. 2, 1976.

A computer-generated hologram named the "inclined-bar" type is proposed for measuring aspheric surfaces. Examples are presented.

10. Sirohi, R. S., H. Blume, and K. J. Rosenbruch, "Optical Testing Using Synthetic Holograms," Optica Acta, Vol. 23, No. 3, 1976.

A modification of the Birch-Green computer generated hologram is presented. The ability of these holograms for evaluating aspherics is demonstrated.

Refer to the following bibliographic entries of Appendix A for additional articles on the applications of computer-generated holograms:

8, 21, 29, 39, 40, 46, 47, 50, 54, 57, 63, 67, 71, 73,
78, 85, 88, 91, 92, 108, 112, 114, 120, 123, 128, 129, 136, 138,
141, 144, 147, 150, 162, 163, 164, 165, 167.

2: A Compiled Bibliography on Computer-
Generated Holography and Related Topics

The following bibliography concerns the subjects of computer-generated holography, spatial filtering, and applications. Some of the articles pertain to classical holography but appear to be applicable to computer-generated holography.

The bibliography is divided into three sections as follows:

(1) articles published in journals, (2) books, and (3) U. S. patents. The entries are listed chronologically except when the month and year of some publications are the same in which case the listing is alphabetical. An alphabetical author's index follows the publications listing with references to the numbered articles of the bibliography.

Some of the bibliographical entries are followed by a statement indicating the source of an abstract on that article.

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Articles

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2. Gabor, D., "Character Recognition by Holography," NATURE, Vol. 208, pp. 422-423, October 30, 1965.
3. Waters, J. P., "Holographic Image Synthesis Utilizing Theoretical Methods," Applied Physics Letters, Vol. 9, No. 11, pp. 405-407, December 1966.

4. Hayes, C., "An Investigation of Real Image Display Spatial Filtering by Means of Holograms," Ph.D. Dissertation, Xerox University Microfilms Order Number 67-10,049, Ann Arbor, Mich., 1966.
Dissertation Abstracts International, B, The Sciences and Engineering, Vol. 28, 1967, p. 1505B.
5. Balstead, J. O., "Holograms and Spatial Filters Processed and Copied in Position," Applied Optics, Vol. 6, No. 1, p. 170, January 1967.
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APPENDIX B

A LISTING OF FORTRAN PROGRAMS

A Listing of FORTRAN Programs

The following programs and subroutines listed in alphabetical order are described in Chapter III. For a complete explanation of the use of these programs, refer to Chapter III.

***** BITS *****

```

SUBROUTINE BITS(A,CLIP,NQ,N12,B)
INTEGER B(4600),NPN(2)
COMPLEX A(64,64)
DATA NPN(1),NPN(2)/0777777777777777.0/
INTEGER QT(2,39)
COMMON/QUAN/QT
NT=QT(1,NQ)
NC=QT(2,NQ)
ACON=FLOAT(NC)/CLIP
ANCON=FLOAT(NC)/6.283185308
DO 1000 I=1,40
  WRITE(9)(NPN(N12),J=1,23)
1000 CONTINUE
NL=NC*23
N2=NC/2
CALL INIT(N2,NC)
DO 2 I=1,NT
DO 3 J=1,NL
B(J)=0
3 CONTINUE
DO 4 J=1,NT
R=AMIN1(REAL(A(I,J)),CLIP)
IMAG=R*ACON+.5
IF(IMAG.EQ.0)GOTO4
ANG=AIMAG(A(I,J))
IANG=ANCON*ANG+.5
IANG=MOD(IANG,NC)
IBL=(J-1)*NC
IWL=IBL/36
ISH=IBL-IWL*36+IANG
IT=ISH/36
IWL=IWL+IT
ISH=ISH-IT*36
CALL BS(B,IWL,IMAG,ISH)
4 CONTINUE
NB=-22
DO 5 I=1,NC
NB=NB+23
NFI=NB+22
DO 7 JJ=NB,NFI
B(JJ)=XOR(B(JJ),NPN(N12))
7 CONTINUE
5 CONTINUE
2 CONTINUE
DO 1001 I=1,40
  WRITE(9)(B(JJ),JJ=NB,NFI)
1001 CONTINUE
RETURN
END

```

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***** BITS2 *****

```

SUBROUTINE BITS2(A,CLIP,NQ,N12)
INTEGER B(4600),NPN(2)
COMPLEX A(64,64)
DATA NPN(1),NPN(2)/0777777777777777,0/
INTEGER QT(2,39)
COMMON/QUAN/QT
NT=QT(1,NQ)
NC=QT(2,NQ)
ACON=FLOAT(NC)/CLIP
ANCON=FLOAT(NC)/6.283185308
DO 1000 I=1,40
WRITE(9)(NPN(N12),J=1,23)
1000 CONTINUE
NL=NC*23
N2=NC
CALL INIT(N2,NC)
DO 2 I=1,NT
DO 3 J=1,NL
B(J)=0
3 CONTINUE
DO 4 J=1,NT
R=AMIN1(REAL(A(I,J)),CLIP)
IMAG=R*ACON*.5
IF(IMAG.EQ.0)GOTO4
ANG=AIMAG(A(I,J))
IANG=ANCON*ANG+.5
IANG=MOD(IANG,NC)
IBL=(J-1)*NC
IWL=IBL/36
ISH=IBL-IWL*36+IANG
IT=ISH/36
IWL=IWL+IT
ISH=ISH-IT*36
CALL BS(B,IWL,IMAG,ISH)
4 CONTINUE
NB=-22
DO 5 I=1,NC
NB=NR+23
NFI=NB+22
DO 7 JJ=NB,NFI
B(JJ)=XOR(B(JJ),NPN(N12))
7 CONTINUE
WRITE(9)(B(JJ),JJ=NB,NFI)
5 CONTINUE
2 CONTINUE
DO 1001 I=1,40
WRITE(9)(NPN(N12),J=1,23)
1001 CONTINUE
RETURN
END

```

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***** BLOCK *****

```
BLOCK DATA
INTEGER GT(2,39)
COMMON/CAH/QT
DATA GT/5,133,6,114,7,100,8,88,9,80,10,72,11,66,12,61,13,57,14,53,
>15,50,16,47,17,44,18,42,19,40,20,38,21,38,22,34,23,33,24,32,
>25,30,26,29,27,28,29,28,31,25,32,24,33,23,34,22,37,21,39,20,
>41,19,43,18,46,17,49,16,52,15,56,14,60,13,64,12/
END
```

***** BS *****

	AXRS
$\$ (1)$	
INIT*	LA A1,*0,X11 ANA,U A1,1 LA A2,SEED LA,U A3,0 DSA A2,0,A1 DS A2,W LA A1,*1,X11 SA A1,N
BS*	J 3,X11 DS AG,RS DS A8,RS+2 DS A10,RS+4 LA A1,*1,X11 LA A2,0,X11 AA A2,A1 DL A6,W LA A3,*3,X11 DSL A6,0,A3 LA A8,W+1 LA,U A9,0 DSA A8,0,A3 OR A7,A8 LA A10,A9 LA A5,*2,X11 LA A4,N ANA A4,A5 SSL A4,1 MSI,U A4,23 AA A2,A4 LA,U A4,0 LXI,U A2,23 AA,U A4,1 OR A6,0,A2 SA A7,0,A2 OR A8,1,A2 SA A9,1,A2 OR A10,2,A2 SA A11,2,*A2 TLE A4,A5 J AGN DL AG,RS DL A8,RS+2 DL A10,RS+4 J 5,X11
$\$ (0)$	+04000000000000
SEED	RES 1
N	RES 2
W	RES 6
RS	ENU

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```

***** CRUPT *****
SUBROUTINE CRUPT(B,C,CLIP,NQ,N)
COMPLEX A(64,64),B(64,64),C(64,64)
INTEGER QT(2,39)
COMMON/QUAN/QT
NT=QT(1,NQ)
NC=QT(2,NQ)
ACON=FLOAT(NC)/CLIP
ANCON=FLOAT(NC)/6.283185308
NL=NT/2+2
NR=(N+N-NT)/2+1
DO 1 I=1,N
DO 1 J=1,N
IF((I.LT.NL.OR.I.GT.NR).AND.(J.LT.NL.OR.J.GT.NR).OR.(NT.GE.N))
>GOTO4
C(I,J)=(0.,0.)
GOTO1
4 R=AMIN1(REAL(B(I,J)),CLIP)
IR=R*ACON+.5
R=FLOAT(IR)/ACON
ANG=AIMAG(B(I,J))
IANG=ANG*ANCON+.5
ANG=FLOAT(IANG)/ANCON
C(I,J)=CMPLX(R*COS(ANG),R*SIN(ANG))
1 CONTINUE
CALL FFT2(C,N)
BSQ=0.
AB=0.
DO 3 I=1,N
DO 3 J=1,N
BMS=REAL(C(I,J))**2+AIMAG(C(I,J))**2
BSQ=BSQ+BMS
AB=AB+REAL(A(I,J))*SQRT(BMS)
3 CONTINUE
SC=AB/BSQ
ERR=(SC*BSQ-2.*AB)*SC+ASQ
ERR=ASQ/ERR
RETURN
END

```

```

***** CRUPT2 *****
*****
***** SUBROUTINE CRUPT2 *****
*****
SUBROUTINE CRUPT2(B,C,CLIP,NQ,N)
COMPLEX A(64,64),B(64,64),C(64,64)
INTEGER QT(2,39)
COMMON/QUAN/QT
NT=QT(1,NQ)
NC=QT(2,NQ)
ACON=FLOAT(NC)/CLIP
ANCON=FLOAT(NC)/6.283185308
NL=NT/2+2
NR=(N+N-NT)/2+1
DO 1 I=1,N
DO 1 J=1,N
IF((I.LT.NL.OR.I.GT.NR).AND.(J.LT.NL.OR.J.GT.NR).OR.(NT.GE.N))
>GOTO4
C(I,J)=(0.,0.)
GOTO1
4 R=AMIN1(REAL(B(I,J)),CLIP)
IR=R*ACON+.5
R=FLOAT(IR)/ACON
ANG=AIMAG(B(I,J))
IANG=ANG*ANCON+.5
ANG=FLOAT(IANG)/ANCON
C(I,J)=CMPLX(R*COS(ANG),R*SIN(ANG))
1 CONTINUE
RETURN
END

```

```
***** ERR *****
FUNCTION ERR(A,B,ASQ,CLIP,NQ,N)
COMPLEX A(64,64),B(64,64),C(64,64)
INTEGER IOT(2,39)
COMMON/QUAN/IOT
NT=IOT(1,NQ)
NC=IOT(2,NQ)
ACON=FLOAT(NC)/CLIP
ANCON=FLOAT(NC)/6.283185308
NL=NT/2+2
NR=(N+M-NT)/2+1
DO 1 I=1,N
DO 1 J=1,N
IF(I.LT.NL.OR.I.GT.NR).AND.(J.LT.NL.OR.J.GT.NR).OR.(NT.GF.N))
>GOTO4
C(I,J)=(0.,0.)
GOTO1
4 R=A MINI(REAL(B(I,J)),CLIP)
IR=R*ACON+.5
R=FLOAT(IR)/ACON
ANG=AIMAG(B(I,J))
IANG=ANG*ANCON+.5
ANG=FLOAT(IANG)/ANCON
C(I,J)=CMPLX(R*COS(ANG),R*SIN(ANG))
1 CONTINUE
CALL FFT2(C,N)
BSQ=0.
AB=0.
DO 3 I=1,N
DO 3 J=1,N
BMS=REAL(C(I,J))**2+AIMAG(C(I,J))**2
BSQ=BSQ+BMS
AB=AB+REAL(A(I,J))*SQRT(BMS)
3 CONTINUE
SC=AB/BSQ
ERR=(SC*BSQ-2.*AB)*SC+ASQ
ERR=ASQ/ERR
RETURN
END
```

***** CODE *****

```
C***** MAIN PROGRAM CODE *****
C***** COMPLEX A(64,64),B(64,64),E(64,64),D(64,64),XX
C***** REAL TE(64)
C***** REAL TT(64)
C***** DO 1000 I=1,64
C***** READ(5,1001) (TE(J),J=1,64)
C***** DO 1000 J=1,64
C***** B(I,J)=CMPLX(TE(J),0.)
1000 CONTINUE
CALL PIC(B,1.,39,64)
DO 1500 I=1,64
READ(5,1001) (TT(J),J=1,64)
DO 1500 J=1,64
D(I,J)=CMPLX(TT(J),0.)
1500 CONTINUE
CALL PIC(D,1.,39,64)
1001 FORMAT(64F1.0)
CALL ROTATE(B,A,64)
CALL ROTATE(D,E,64)
CALL FFT2(A,64)
CALL FFT2(E,64)
DO 2000 I=1,64
DO 2000 J=1,64
XX=A(I,J)-E(I,J)
A(I,J)=XX
2000 CONTINUE
CALL POLAR(A,B,64)
CALL ROTATE(B,A,64)
DO 1 I=1,3
READ(5,100) NQ,PCT
FORMAT(I2,F5.2)
100 CLIP=PCT*FMAX(A,64)
CALL BUILD(A,B,NQ,64)
CALL BITS(B,CLIP,NQ,2)
CALL PIC(B,CLIP,NQ,64)
1 CONTINUE
STOP
END FILE 9
END
```

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***** FMAX *****

```

FUNCTION FMAX(A,N)
COMPLEX A(64,64)
SUM=0.
DO 1 I=1,N
DO 1 J=1,N
IF(REAL(A(I,J)).GT.SUM)SUM=REAL(A(I,J))
CONTINUE
FMAX=SUM
RETURN
END

```

***** HOLPLO *****

```

***** MAIN PROGRAM HOLPLO *****
***** COMPLEX A(64,64),B(64,64)
REAL TE(64)
DIMENSION C(4600)
DO 1000 I=1,64
READ(5,1001)(TE(J),J=1,64)
1001 FORMAT(64E1.0)
DO 1000 J=1,64
B(I,J)=CMPLX(TE(J),0.)
1000 CONTINUE
CALL PIC(B,1.,39,64)
CALL ROTATE(B,A,64)
WRITE(6,98)
CALL FFT2(A,64)
DO 999 I=1,64
DO 999 J=1,64
A(I,J)=CONJG(A(I,J))
CALL POLAR(A,B,64)
CALL ROTATE(B,A,64)
DO 1 I=1,3
READ(5,100)NQ,PCT
100 FORMAT(I2,F5.2)
CLIP=PCT*FMAX(A,64)
CALL BUILD(A,B,NQ,64)
CALL BITS(B,CLIP,NQ,1,C)
WRITE(6,98)
98 FORMAT('1')
99 FORMAT(4(6X,2G11.4))
CALL PIC(B,CLIP,NQ,64)
1 CONTINUE
END FILE 9
STOP
END

```

```

***** FFT2 *****
SUBROUTINE FFT2(A,N)
INTEGER GAMMA,S,G,U,D
COMPLEX A(64,64),B,W(64),Z(64)
GAMMA=ALOG(FLOAT(N))/.6931471805+.1
M=N/2
PHI=6.283185308/FLOAT(N)
DO 91 J=1,M
THET=PHI*FLOAT(J-1)
W(J)=CMPLX(COS(THET),-SIN(THET))
91 CONTINUE
DO 200 IR=1,N
DO 20 I=1,N
Z(I)=A(I,IR)
20 CONTINUE
S=1
D=M
DO 100 J=1,GAMMA
DO 30 L1=1,S
L1CON=(L1-1)*D
DO 30 L=L1CON
I=L+L1CON
IM=I+M
K=MOD(L+L1CON*2,N)
U=K+D
G=L1CON+1
B=Z(U)*W(G)
A(I,IR)=Z(K)+B
A(IM,IR)=Z(K)-B
30 CONTINUE
S=S*2
D=D/2
DO 100 JJ=1,N
Z(JJ)=A(JJ,IR)
100 CONTINUE
200 CONTINUE
M=N/2
DO 1200 IR=1,N
DO 1020 I=1,N
Z(I)=A(IR,I)
1020 CONTINUE
S=1
D=M
DO 1100 J=1,GAMMA
DO 1030 L1=1,S
L1CON=(L1-1)*D
DO 1030 L=L1CON
I=L+L1CON
IM=I+M
K=MOD(L+L1CON*2,N)
U=K+D
G=L1CON+1
B=Z(U)*W(G)
A(IR,I)=Z(K)+B
A(IR,IM)=Z(K)-B
1030 CONTINUE
S=S*2
D=D/2
DO 1100 JJ=1,N
Z(JJ)=A(IR,JJ)
1100 CONTINUE
1200 CONTINUE
RETURN
END

```

```

**** OPT *****
      SUBROUTINE OPT(A,B,F,NQ,PCT,X)
C THIS SUBROUTINE FINDS THE OPTIMUM S/N RATIO IN TERMS OF CLIPPING PARAMETER
C PCT AND BANDWIDTH PARAMETER NQ. A AND B ARE COMPLEX MATRICES - A IS
C A REFERENCE FFT(FFT(A)) IN POLAR FORM ; WHILE B IS FFT(B) IN POLAR FORM.
C COMPLEX A(64,64), B(64,64)
      REAL F(13,9)
      CL=MAX(B,64)/9.
      ASQ=0.
      DO 1 I=1,64
      DO 1 J=1,64
      ASQ= ASQ + REAL(A(I,J))**2
1     CONTINUE
      NQ=39
      I=NQ/3
      Z=0.
      X=0.
      TOL=0.001
      KT=0
30    CONTINUE
      DO 5 J=1,9
      NQ= 3*I
      CLIP=CL*FLOAT(J)
      F(I,J)=ERR(A,B,ASQ,CLIP,NQ,64)
      IF(F(I,J).LE.X) GO TO 5
      X=F(I,J)
      J1=J
      IF(ABS(X-Z).GT.TOL) GO TO 5
      GO TO 25
5     CONTINUE
      WRITE(6,110) NQ,J1,X
      J=J1
      DO 10 I=1,13
      NQ=3*I
      CLIP=CL*FLOAT(J)
      F(I,J)=ERR(A,B,ASQ,CLIP,NQ,64)
      IF(F(I,J).LE.Z) GO TO 10
      Z=F(I,J)
      I1=I
      NQ1=3*I1
      IF(ABS(X-Z).GT.TOL) GO TO 10
      GO TO 25
10   CONTINUE
      WRITE(6,110) NQ1,J1,Z
      I=I1
      GO TO 30
25   IF((I1.EQ.1).OR.(I1.EQ.13)) GO TO 35
50   IF((J1.EQ.1).OR.(J1.EQ.9)) GO TO 55
      GO TO 40
35   IF(I1.EQ.1) GO TO 45
      I1=I1-1
      KT=KT+1
      GO TO 50
45   I1=I1+1
      KT=KT+1
      GO TO 50
55   IF(J1.EQ.1) GO TO 60
      J1=J1-1
      KT=KT+1
      GO TO 40
60   J1=J1+1
      KT=KT+1
40   N=I1+1
      M=I1-1
      DO 20 I=M,N
      L=J1-1
      K=J1+1
      DO 20 J=L,K
      NQ=3*I
      CLIP=CL*FLOAT(J)
      IF(KT.GT.0) GO TO 70
      IF((I.EQ.I1).AND.(J.EQ.J1)) GO TO 20
70   CONTINUE
      F(I,J)=ERR(A,B,ASQ,CLIP,NQ,64)
      WRITE(6,115) I,J,F(I,J)
      IF(F(I,J).LE.X) GO TO 65
      X=F(I,J)
      I1=I
      J1=J
      GO TO 25
65   20 CONTINUE
15    CONTINUE
      WRITE(6,99)

```

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```

***** OPT *****
99  FORMAT(//,' THE OPTIMUM BANDWIDTH AND CLIPPING LEVELS
C',//)
      WRITE(6,100) I1,J1
      NQ=NQ-3
      PC1=FLOAT(J1)/9.
      WRITE(6,105) NQ,PCT,X
110  FORMAT(2X,'NQ=',I2,3X,'CL=',I2,3X,'S/N=',G12.6,//)
100  FORMAT(6X,'NQ',6X,'CLIP',9X,'F',I2,'',I2,'')',//)
105  FORMAT(6X,I2,6X,G8.4,3X,G12.6)
115  FORMAT(2X,'I=',I2,3X,'J=',I2,3X,'S/N=',G12.6,//)
      RETURN
      END

***** OPT1 *****
COMPLEX A(64,64),B(64,64)
REAL TE(64)
REAL F(13,9)
DO 1000 I=1,64
READ(5,1001) (TE(J),J=1,64)
1001 FORMAT(64F1.0)
DO 1000 J=1,64
A(I,J)= CMPLX(TE(J),0.)
1000 CONTINUE
CALL FFT2(A,64)
CALL POLAR(A,B,64)
CALL FFT2(A,64)
CALL POLAR(A,A,64)
CALL OPT(A,B,F,NQ,PCT,X)
STOP
END.

```

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```

***** OPT2 *****
C***** MAIN PROGRAM OPT2 *****
C***** COMPLEX A(64,64),B(64,64),CONJG
C***** COMPLEX E(64,64),D(64,64),XX
C***** REAL TT(64)
C***** REAL TE(64)
C***** REAL F(13,9)
C***** DO 1000 I=1,64
C***** READ(5,1001) (TE(J),J=1,64)
1001 FORMAT(64F1.0)
C***** DO 1000 J=1,64
C***** A(I,J)=CMPLX(TE(J),0.)
1000 CONTINUE
C***** DO 1500 I=1,64
C***** READ(5,1001) (TT(J),J=1,64)
C***** DO 1500 J=1,64
C***** E(I,J)=CMPLX(TT(J),0.)
1500 CONTINUE
C***** CALL FFT2(A,64)
C***** CALL FFT2(E,64)
C***** DO 2000 I=1,64
C***** DO 2000 J=1,64
C***** RR=REAL(E(I,J))
C***** PP=-AIMAG(E(I,J))
C***** D(I,J)=CMPLX(RR,PP)
C***** SS=CABS(D(I,J))**2
C***** IF(SS.LT.0.001) SS=0.001
C***** XX=A(I,J)*D(I,J)/SS
C***** A(I,J)=XX
2000 CONTINUE
C***** CALL POLAR(A,B,64)
C***** CL=FMAX(B,64)/9.
C***** CALL FFT2(A,64)
C***** CALL POLAR(A,A,64)
C***** ASQ=0.
C***** DO 1 I=1,64
C***** DO 1 J=1,64
1 ASQ= ASQ + REAL(A(I,J))**2
CONTINUE
NQ=39
I=NQ/3
Z=0.
X=0.
TOL=0.001
KT=0
30 CONTINUE
DO 5 J=1,9
NQ= 3*I
CLIP=CL*FLOAT(J)
F(I,J)=ERR(A,B,ASQ,CLIP,NQ,64)
IF(F(I,J).LE.Z) GO TO 5
X=F(I,J)
J1=J
IF(ABS(X-Z).GT.TOL) GO TO 5
GO TO 25
5 CONTINUE
WRITE(6,110) NQ,J1,X
JEJI
DO 10 I=1,13
NQ=3*I
CLIP=CL*FLOAT(J)
F(I,J)=ERR(A,B,ASQ,CLIP,NQ,64)
IF(F(I,J).LE.Z) GO TO 10
Z=F(I,J)
I1=I
NQ1=3*I1
IF(ABS(X-Z).GT.TOL) GO TO 10
GO TO 25
10 CONTINUE
WRITE(6,110) NQ1,J1,Z
I=I1
GO TO 30
25 IF({I1.EQ.1}:OR:{I1.EQ.13}) GO TO 35
IF({J1.EQ.1}:OR:{J1.EQ.9}) GO TO 55
GO TO 40
35 IF(I1.EQ.1) GO TO 45
I1=I1-1

```

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```

***** OPT2 *****
      KT=KT+1
      GO TO 50
 45    I1=I1+1
      KT=KT+1
      GO TO 50
 55    IF(J1.EQ.1) GO TO 60
      J1=J1-1
      KT=KT+1
      GO TO 40
 60    J1=J1+1
      KT=KT+1
 40    N=I1+1
      M=I1-1
      DO 20 I=M,N
      L=J1-1
      K=J1+1
      DO 20 J=L,K
      NQ=3*I
      CLIP=CLIP+FLOAT(J)
      IF(KT.GT.0) GO TO 70
      IF((I.EQ.11).AND.(J.EQ.J1)) GO TO 20
 70    CONTINUE
      F(I,J)=ERR(A,B,A5Q,CLIP,NQ,64)
      WRITE(6,115) I,J,F(I,J)
      IF(F(I,J).LE.X) GO TO 65
      X=F(I,J)
      I1=I
      J1=J
      GO TO 25
 65    IF(KT.GE.3) GO TO 15
 20    CONTINUE
 15    CONTINUE
 99    WRITE(6,99),
      FORMAT(//,' THE OPTIMUM BANDWIDTH AND CLIPPING LEVEL VALUES
      C',//),
      WRITE(6,100) I1,J1
      NQ=NQ-3
      CLIP= FLOAT(J1)/FLOAT(I1)
      WRITE(6,105) NQ,CLIP,X
 110   FORMAT(2X,'NQ=',I2,3X,'CLIP=',I2,3X,'S/N=',G12.6,//)
 100   FORMAT(6X,'NQ',6X,'CLIP',9X,'F(',I2,',',I2,')+',//)
 105   FORMAT(6X,I2,6X,G8.4,3X,G12.6)
 115   FORMAT(2X,'I=',I2,3X,'J=',I2,3X,'S/N=',G12.6,//)
      STOP
      END

```

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***** PIC *****

```

SUBROUTINE PIC(A,CLIP,NQ,N)
COMPLEX A(64,64)
INTEGER IG(64)
INTEGER QT(2,39)
COMMON/QUAN/QT
NT=QT(1,NQ)
NQ=QT(2,NQ)
WRITE(6,101)NQ,NT,NC,CLIP
101 FORMAT('1',3I10,G10.5)
ACON=FLOAT(NC)/CLIP
DO 1 I=1,NT
DO 2 J=1,NT
IG(J)=AMIN1(REAL(A(I,J)),CLIP)*ACON+.5
2 CONTINUE
WRITE(6,100)(IG(J),J=1,NT)
100 FORMAT(' ',64I2)
1 CONTINUE
RETURN
END

```

***** POLAR *****

```

SUBROUTINE POLAR(A,B,N)
COMPLEX A(64,64),B(64,64)
DO 1 I=1,N
DO 1 J=1,N
BR=CABS(A(I,J))
IF(BR.GT.0.)GOTO2
ANG=0.
GOTO 3
2 ANG=ATAN2(AIMAG(A(I,J)),REAL(A(I,J)))
IF(ANG.LT.0.)ANG=ANG+6.283185308
3 B(I,J)=CMPLX(BR,ANG)
1 CONTINUE
RETURN
END

```

***** BUILD *****

```

SUBROUTINE BUILD(A,B,NQ,N)
COMPLEX A(64,64),B(64,64)
INTEGER QT(2,39)
COMMON/QUAN/QT
NT=QT(1,NQ)
NQ=QT(2,NQ)
NL=N/2-(NT+1)/2+1
NR=N/2+NT/2
IN=0
DO 1 I=NL,NR
IN=IN+1
JN=0
DO 1 J=NL,NR
JN=JN+1
B(IN,JN)=A(I,J)
1 CONTINUE
RETURN
END

```

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```

***** RECON *****
      GO TO 13
16     ANL=FLOAT(NL)
      THET=3*PI/2-ANL*2*PI/NC
      GO TO 17
11     ANL=FLOAT(NL)
      THET=ANL*2*PI/NC-PI/2
      GO TO 17
12     ANR=FLOAT(NR)
      THET=PI/2+ANR*2*PI/NC
      GO TO 17
13     ANR=FLOAT(NR)
      THET=450*PI/180-ANR*2*PI/NC
      IF(ANR.EQ.0.) THET=0.
      GO TO 17
17     G(I0,I1) = THET
      ID=NC/2+NC*11
      IL=NC/2-1+NC*11
      IX=IL-NC/2+1
      IUX=ID+NC/2-1
20     CONTINUE
15     CONTINUE
55     CONTINUE
92     FORMAT(1X,'F',1X,23I5)
16     CONTINUE
      DO 61 I=1,40
61     READ(9) IN
      DO 43 I=1,NT
43     WRITE(6,91) (E(I,J),J=1,NT)
      DO 44 I=1,NT
44     WRITE(6,93) (G(I,J),J=1,NT)
      93    FORMAT(1X,'ANGLE',1X,32F3.1)
      DO 42 I=1,NT
      DO 42 J=1,NT
42     D(I,J)=CPLX(E(I,J),G(I,J))
      FORMAT(1X,2G4.1)
      DO 40 I=1,NT
      DO 40 J=1,NT
      RE=E(I,J)*COS(G(I,J))
      AIM=E(I,J)*SIN(G(I,J))
      BT(I,J)=CMPLX(RE,AIM)
40     CONTINUE
91     FORMAT(1X,' E',1X,64I1)
108    FORMAT(1X,' E',1X,214)
      CALL ROTATE(B,A,64)
      DO 600 I=1,64
600    READ (9) (E(I,J),J=1,64)
      DO 601 I=1,64
      DO 601 J=1,64
      XX=A(I,J)*E(I,J)
      A(I,J)=XX
      CALL FFT2(A,64)
      CALL POLAR(A,B,64)
      CALL ROTATE(B,A,64)
      NG=39
      PCT=.25
      CLIP=PCT+FMNAX(A,64)
      CALL BUILD(A,B,NQ,64)
      CALL BITS2(B,CLIP,NQ,2)
      WRITE(6,98)
98     FORMAT('1')
      CALL PIC(B,CLIP,NQ,64)
500    CONTINUE
      ENDFILE 9
      STOP
      END

```

```

***** ROTATE *****
SUBROUTINE ROTATE(A,B,N)
COMPLEX A(64,64),B(64,64)
NA=N/2-2
DO 1 I=1,N
DO 1 J=1,N
IN=MOD(I+NA,N)+1
JN=MOD(J+NA,N)+1
B(IN,JN)=A(I,J)
1 CONTINUE
RETURN
END

```

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```

***** SIMUL *****
*****
C ***** MAIN PROGRAM SIMUL
C ****
DIMENSION U(64)
DIMENSION V(64)
COMPLEX D(64,64),E(64,64)
COMPLEX C(64,64)
COMPLEX A(64,64),B(64,64),CONJG
REAL TE(64)
DO 1000 I=1,64
READ(S,1001)(TE(J),J=1,64)
DO 1000 J=1,64
B(I,J)=CMPLX(TE(J),0.)
1000 CONTINUE
1001 FORMAT(64F1.0)
CALL PIC(B,1:,39,64)
CALL ROTATE(B,A,64)
WRITE(6,98)
CALL FFT2(A,64)
CALL POLAR(A,B,64)
READ(5,100)NQ,PCT
CALL ROTATE(B,D,64)
CALL BUILD(D,E,NQ,64)
DO 41 I=1,64
DO 411 J=1,4
V(J)=REAL(E(I,J))
U(J)=AIMAG(E(I,J))
IF(V(J).LT.-1) U(J)=0.
411 CONTINUE
WRITE(6,107) (U(J),J=1,64)
41 CONTINUE
107 FORMAT(IX,' ANGLE',32F3.1)
CLIP=PCT*FMAX(B,64)
CALL CRUPT(B,C,CLIP,NQ,64)
CALL POLAR(C,B,64)
CALL ROTATE(B,A,64)
PCT=1.
NQ=39
CLIP=PCT*FMAX(A,64)
CALL BITS2(A,CLIP,NQ,2)
100 FORMAT(I2,F5.2)
WRITE(6,98)
98 FORMAT('1')
99 FORMAT(4(6X,2G11.4))
CALL PIC(A,CLIP,NQ,64)
1 CONTINUE
END FILE 9
STOP
END

***** SN *****
*****
C ***** MAIN PROGRAM SN
C ****
COMPLEX A(64,64),B(64,64)
REAL TE(64)
REAL SN(9)
COMMON/QUAN/QT(2,39)
DO 1000 I=1,64
READ(5,1001)(TE(J),J=1,64)
1001 FORMAT(64F1.0)
DO 1000 J=1,64
A(I,J)=CMPLX(TE(J),0.)
1000 CONTINUE
CALL FFT2(A,64)
CALL POLAR(A,B,64)
CL=FMAG(B,64)/9.
CALL FFT2(A,64)
CALL POLAR(A,A,64)
ASQ=0.
DO 1 I=1,64
DO 1 J=1,64
ASQ=ASQ+REAL(A(I,J))**2
1 CONTINUE
DO 2 NQ=3,39,3
WRITE(6,101)NQ,QT(1,NQ),QT(2,NQ)
101 FORMAT(//,',',3I5)
DO 3 I=1,9
CLIP=CL*FLOAT(I)
SN(I)=ERR(A,B,ASQ,CLIP,NQ,64)
3 CONTINUE
WRITE(6,100)(SN(I),I=1,9)
100 FORMAT('0',9G12.6)
2 CONTINUE
STOP
END

```

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***** SNPLOT *****
C***** MAIN PROGRAM SNPLOT
C***** MUST BE RUN SEQUENTIALLY AFTER HOLPLO
C      COMPILER(FLEABIS)
PARAMETER NC=13,NT=43,MIO=33
DIMENSION G(NT,NT)
INTEGER F(MC,NT),E(NT,NT)
COMPLEX A(64,64),B(64,64),D(64,64)
REAL PI(23)
PI=3.1415926
1    CONTINUE
DO 60 I=1,40
60    READ(9) IN
200   CONTINUE
DO 10 I=1,NT
NC=IC
MC=(IC-1)
JX=0
DO 50 JR=M,N
NR=M+1
CONTINUE
IF (I0.EQ.IUX) GO TO 9
ID=ID+1
XCHECK
GO TO 3
9    IF (NR.LT.NC/4) NR=0
IF (NL.EQ.1A).AND.(NR.EQ.0) GO TO 16
GO TO 13
16    ANL=FLOAT(IL)
THET=3*PI/2-ANL*2*PI/NC
GO TO 17
11    ANL=FLOAT(IL)
THET=ANL*2*PI/NC-PI/2
GO TO 17
12    ANR=FLOAT(NR)
THET=PI/2+ANR*2*PI/NC
GO TO 17
13    ANR=FLOAT(NR)
THET=450*PI/180-ANR*2*PI/NC
IF (ANR.EQ.0.) THET=0.
GO TO 17
17    G(1G,IC)=THET
IG=NC/2+NC+II
IB=NC/2-1+NC+II
IUX=IB-NC/2+1
IX=IB+NC/2-1
20    CONTINUE
25    CONTINUE
55    CONTINUE
92    FORMAT(1X,'F',1X,23I5)
10    CONTINUE
DO 61 I=1,40
61    READ(9) IN
DO 44 I=1,NT
44    WRITE(6,93) (G(I,J),J=1,NT)
93    FORMAT(1X,'ANGLE',1X,3ZF3.1)
DO 42 I=1,NT
DO 42 J=1,NT
42    O(I,J)=CMPLX(E(I,J),G(I,J))
FORMAT(1X,26F4.1)
DO 40 I=1,NT
DO 40 J=1,NT
RE=F(I,J)+COS(G(I,J))
AIM=S(I,J)*IN(G(I,J))
B(I,J)=CMPLX(RE,AIM)
40    CONTINUE
DO 999 I=1,64
999   READ(9) (D(I,J),J=1,64)
CONTINUE
READ(9) FB
READ(9) FD
Z=FD/FIAX(B,64)
WHITE(6,88) Z,FD,FB
68    FORMAT(5X,'CLIP=FMAXB*BCT=1,1F10.4,5X,'FMAXD=1F10.4,5X,'FMAXB=1
1,1F10.4,/)
CALL FFT2(B,64)
CALL POLAR (B,B,64)
CLIP=FMAX(B,64)
CALL POLAR(B,B,NT)
CALL ROTATE(B,B,NT)
ASC=0.
DO 1001 I=1,NT
DO 1001 J=1,NT

```

***** SNPLOT *****

```

      JX=JX+1
50   DO 50 JC=1,NT
      F(JX,JC)=0
      JX=0
      DO 55 JL=M,N
      JX=JX+1
      READ(9) IN
      IC=0
      DO 15 J=1,23
      NA=NC/2
      IH=(NC/2-1)
      ID=NC/2
      IBX=0
      IDX=NC-1
      IS=NC/2
      X=FLOAT(NA)
      XCK=0.
      CK=0.
      Z=X-1.
      JJ=39/NC
      DO 15 II=1,JJ
      IC=IC+1
      IF((IC.GT.11)) GO TO 15
      F(JX,IC)=FLD(IS,1,IN(J))
      IF(F(JX,IC).NE.0) GO TO 103
      IS=IS-3
      F(JX,IC)=FLD(IS,1,IN(J))
      IF(F(JX,IC).NE.0) GO TO 102
      IS=IS+7
      F(JX,IC)=FLU(IS,1,IN(J))
      IS=IS-4
      GO TO 103
102  IS=IS+3
103  CONTINUE
      E(IG,IC)=E(IG,IC)+F(JX,IC)
      IS=IS+NC
      OK=FLOAT(E(IG,IC))
      IF(OK.EQ.0) GO TO 20
      CI=FLOAT(N-JL)
      IF((CI.EQ.2).AND.(XCK.LT..0001)) GO TO 21
      IF(X.NE.CT) GO TO 20
21   CONTINUE
      NL=0
      NK=0
      XCK=0.
      4   CONTINUE
      CK=FLU(IB,1,IN(J))
      IF((XCK.EQ.1.).AND.(CK.EQ.0.)) GO TO 16
      IF((CK.EQ.0.)) GO TO 14
      NL=NL+1
      IF(NL.EQ.NA) GO TO 7
      14  CONTINUE
      IF(IR.EQ.IBX) GO TO 5
      IB=IR-1
      XCK=CK
      GO TO 4
      5   IF(NL.LE.NC/4) GO TO 2
      GO TO 11
2     NL=0
      XCK=0.
      7   CONTINUE
      CK=FLU(ID,1,IN(J))
      IF((XCK.EQ.1.).AND.(CK.EQ.0.)) GO TO 12
      IF(CK.EQ.0.) GO TO 3
      ASQ=ASQ+REAL(D(I,J))*2
1001  CONTINUE
      IJ=39
      SNEPR(D,B,ASQ,CLIP,NO,64)
      WRITE(6,600) SN
      FORMAT('0'//,' SN='16I2.6,/)
      ENDFILE 9
      STOP
      END

```

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***** SPAFIL *****

* * * * * TAPE * * * * *

```

SUBROUTINE TAPE(L,$)
INTEGER L(1)
DATA MOD/0776000000000/
READ(9,END=99)(L(I),I=1,23)
L(23)=AND(L(23),MOD)
RETURN
99 RETURN 2
END

```

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